MODULE-2

Network Theorems: Super Position theorem, Thevenin's theorem, Norton's theorem, and Maximum power transfer theorem. (Problems with independent AC and DC sources only).

Teaching-Learning Process

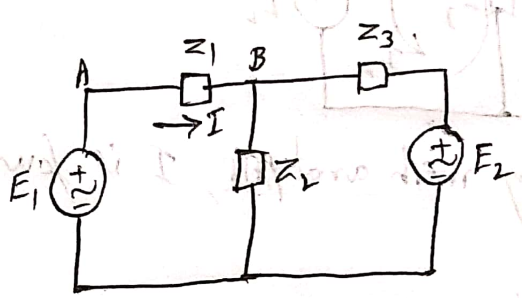
Superposition theorem

Q. State, explain and prove superposition theorem.

Sol: <u>Statement</u>: In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as algebraic sum of the individual contributions of each source acting alone.

When determining the contribution due to a particular independent source, we disable all the remaining independent sources. That is, all the remaining voltage sources are short circuited and current sources are open circuited. The dependent sources are unaltered;

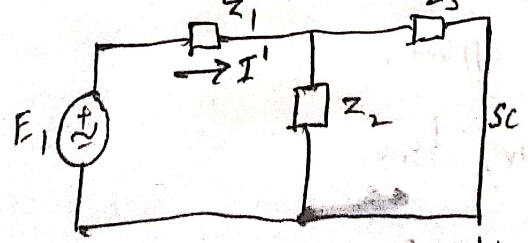
<u>Explanation</u>:



Consider a network, shown in the above figure, having two voltage sources E_1 & E_2.

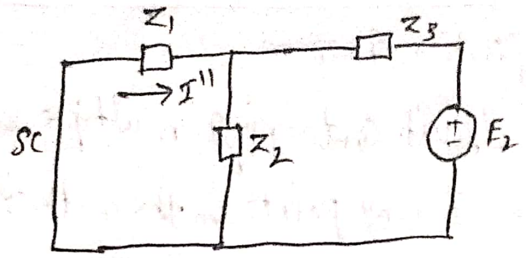
The current (I) in the branch has to be calculated using superposition theorem.

case (i) <u>Source E_1 is acting alone</u>, <u>E_2 is short circuited</u>.



By any of the network reduction techniques, the current I' is found.

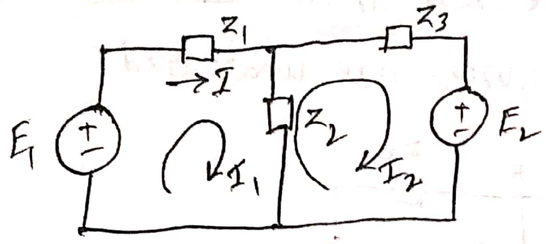
Case (ii) Source E_2 is acting alone, E_1 is short circuited.



By any of the network reduction techniques, the current I'' is found.

According to superposition theorem, the current through the branch AB is given by
$$I = I' + I''$$

Proof: Consider the network shown below.



In general, using mesh analysis, I is found.

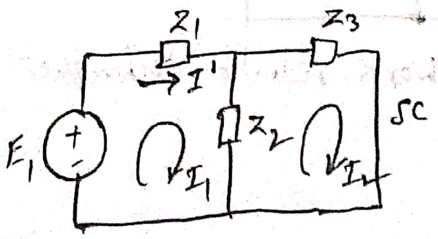
By inspection method,

$$(Z_1+Z_2)I_1 + (-Z_2)I_2 = E_1$$
$$(-Z_2)I_1 + (Z_2+Z_3)I_2 = -E_2$$

$$I = I_1 = \frac{\begin{vmatrix} E_1 & -Z_2 \\ -E_2 & (Z_2+Z_3) \end{vmatrix}}{\begin{vmatrix} (Z_1+Z_2) & -Z_2 \\ -Z_2 & (Z_2+Z_3) \end{vmatrix}} = \frac{E_1 Z_2 + E_1 Z_3 - E_2 Z_2}{Dr} \quad -(1)$$

Now, the superposition theorem is applied.

Case (i) Source E_1 acting alone

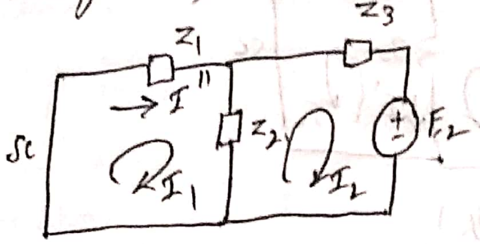


By inspection method,

$$(Z_1+Z_2)I_1 + (-Z_2)I_2 = E_1$$
$$(-Z_2)I_1 + (Z_2+Z_3)I_2 = 0$$

$$I' = I_1 = \frac{\begin{vmatrix} E_1 & -Z_2 \\ 0 & (Z_2+Z_3) \end{vmatrix}}{\begin{vmatrix} (Z_1+Z_2) & -Z_2 \\ -Z_2 & (Z_2+Z_3) \end{vmatrix}} = \frac{E_1 Z_2 + E_1 Z_3}{D_r} \quad -②$$

Case (ii) Source E_2 acting alone,



By inspection method,

$$(Z_1+Z_2)I_1 + (-Z_2)I_2 = 0$$
$$(-Z_2)I_1 + (Z_2+Z_3)I_2 = -E_2$$

$$I'' = I_1 = \frac{\begin{vmatrix} 0 & -Z_2 \\ -E_2 & (Z_2+Z_3) \end{vmatrix}}{\begin{vmatrix} (Z_1+Z_2) & -Z_2 \\ -Z_2 & (Z_2+Z_3) \end{vmatrix}} = \frac{-E_2 Z_2}{D_r} \quad -③$$

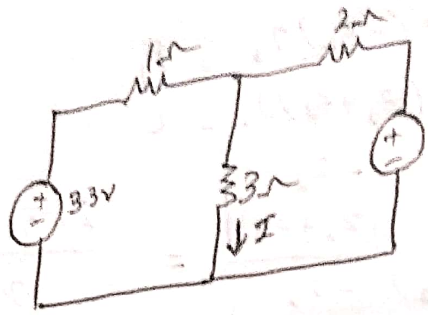
$$\underset{Eq②+Eq③}{I = I' + I''} = \frac{E_1 Z_2 + E_1 Z_3}{D_r} - \frac{E_2 Z_2}{D_r} = \frac{E_1 Z_2 + E_1 Z_3 - E_2 Z_2}{D_r} \quad -④$$

Comparing equations ① & ④, mesh analysis & superposition theorem results are same, Hence superposition theorem is verified (proved).

Q. What are the limitations of superposition theorem.

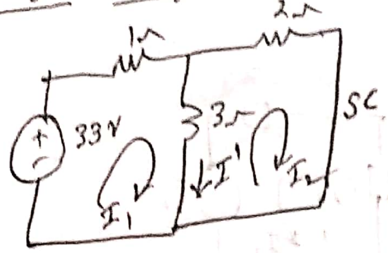
Sol: Superposition is a fundamental property of linear equations and therefore, can be applied to any effect that is linearly related to the source. That is, superposition principle applies only to the current and voltage in a linear circuit but it cannot be used to determine power because power is a non-linear function.

Q. Find current in 3Ω resistance by superposition theorem for the circuit shown in figure.



Sol:

Source 33 V acting alone



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(Apply inspection method only when there are no current sources in the given problem)

By inspection method,

$$(4)I_1 + (-3)I_2 = 33$$

$$(-3)I_1 + (5)I_2 = 0$$

Using calculator,
$a_1 = 4, b_1 = -3$
$c_1 = 33$
$a_2 = -3, b_2 = 5$
$c_2 = 0$

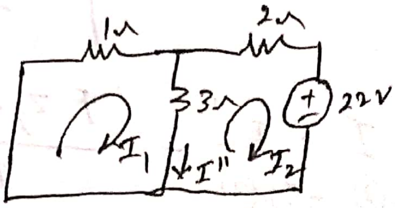
$$I_1 = 15A$$

$$I_2 = 9A$$

$$I' = I_1 - I_2 = 15 - 9$$

$$I' = 6A$$

Source 22V acting alone:



By inspection method

$$(4)I_1 + (-3)I_2 = 0$$

$$(-3)I_1 + (5)I_2 = -22$$

$$I_1 = -6A$$

$$I_2 = -8A$$

$$I'' = I_1 - I_2 = (-6) - (-8)$$

$$I'' = 2A$$

By superposition theorem, $I = I' + I'' = 6 + 2 = 8A$

Note: Take the direction of current I' & I'' in the same direction as of current I

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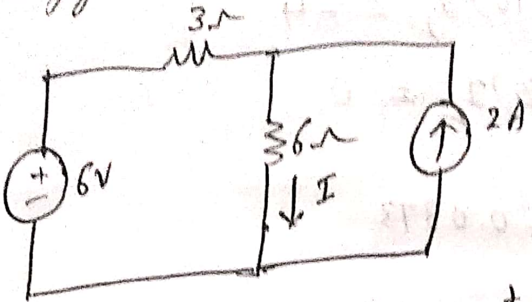
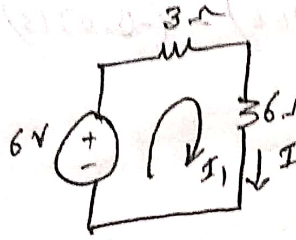
Q. Find the current in the 6Ω resistor using superposition theorem for the circuit shown in figure.

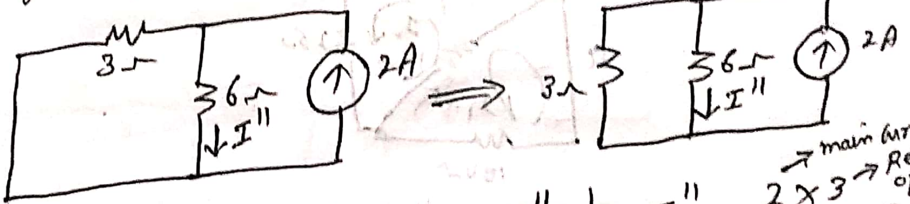


Sol: Source 6V acting alone, 2A current source is open circuited



$$I' = I_1 = \frac{6}{(3+6)} = 0.666A$$

Source 2A acting alone, 6V voltage source is short circuited.



By current division method, $I'' = \frac{2 \times 3}{(6+3)} = 0.666A$

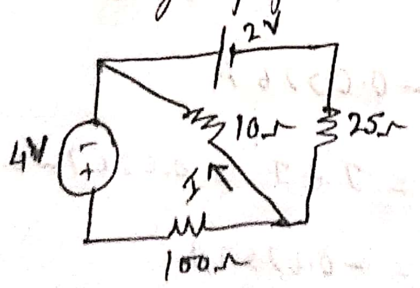
→ main current
→ Resistance of opposite branch
sum of two branch resistances.

By superposition theorem, $I = I' + I''$

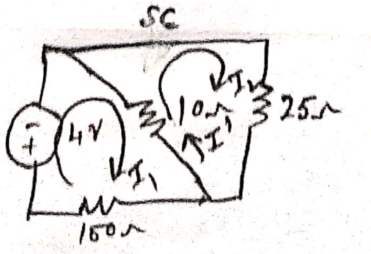
$I = 0.666 + 0.666$

$I = 1.332A$

Q. Find the current I using superposition theorem for the circuit shown in figure.



Sol: Source 4V acting alone,



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By inspection method,

$$(110)I_1 + (-10)I_2 = -4$$

$$(-10)I_1 + (35)I_2 = 0$$

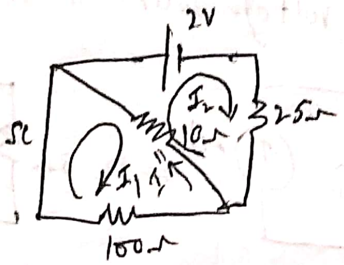
$$I_1 = -0.0373$$

$$I_2 = -0.0106$$

$$I' = I_2 - I_1 = (-0.0106) - (-0.0373)$$

$$I' = 0.0267\text{ A}$$

Soure 2V acting alone,



By inspection method,

$$(110)I_1 + (-10)I_2 = 0$$

$$(-10)I_1 + (35)I_2 = -2$$

$$I_1 = -5.33\text{ mA}$$

$$I_2 = -0.058\text{ A}$$

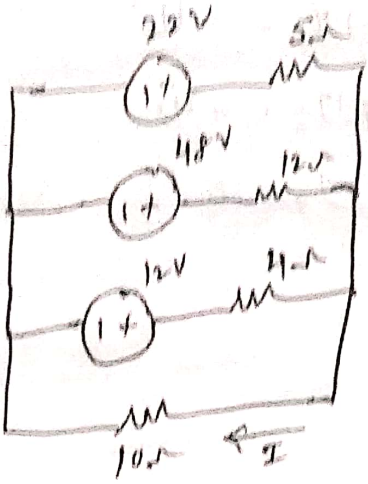
$$I'' = I_2 - I_1 = (-0.058) - (-5.33\times10^{-3})$$

$$I'' = -0.0526\text{ A}$$

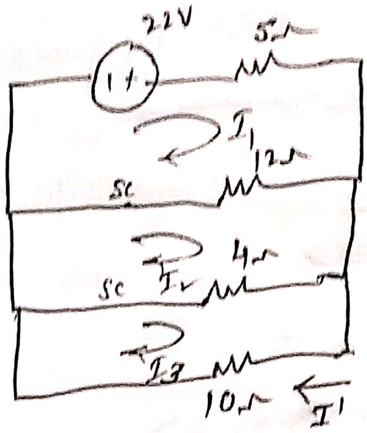
By superposition theorem, $I = I' + I'' = 0.0267 - 0.0526$

$$I = -0.0259\text{ A}$$

Q. Find the current through 10Ω for the circuit shown in figure using superposition theorem.



Sol: 22V source acting alone,

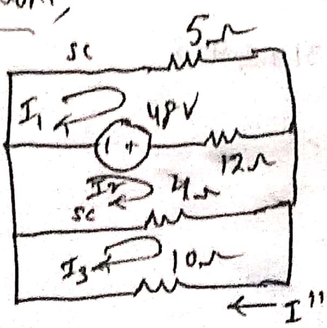


By inspection method,

$$(17)I_1 + (-12)I_2 + (-0)I_3 = 22$$
$$(-12)I_1 + (16)I_2 + (-4)I_3 = 0$$
$$(-0)I_1 + (-4)I_2 + (14)I_3 = 0$$

$$I_1 = 3.01A$$
$$I_2 = 2.43A$$
$$I' = I_3 = 0.694A$$

48V Source acting alone,

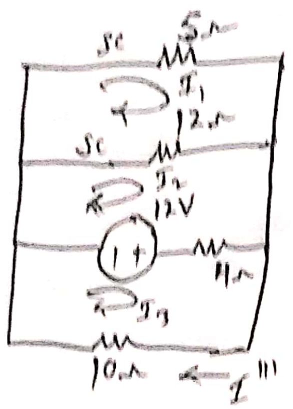


By inspection method,

$$(17)I_1 + (-12)I_2 + (-0)I_3 = -48$$
$$(-12)I_1 + (16)I_2 + (-4)I_3 = +48$$
$$(-0)I_1 + (-4)I_2 + (14)I_3 = 0$$

$$I_1 = -1.263\,A$$
$$I_2 = 2.210\,A$$
$$I'' = I_3 = 0.631\,A$$

<u>12V Source acting alone,</u>



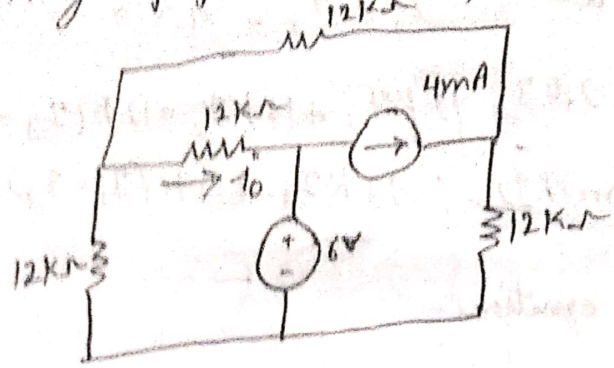
$$(17)I_1 + (-12)I_2 + (-0)I_3 = 0$$
$$(-12)I_1 + (16)I_2 + (-4)I_3 = -12$$
$$(0)I_1 + (-4)I_2 + (14)I_3 = +12$$

$$I_1 = -0.947\,A$$
$$I_2 = -1.342\,A$$
$$I''' = I_3 = 0.473\,A$$

By superposition theorem, $I = I' + I'' + I'''$

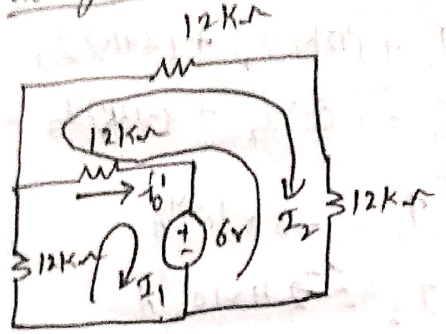
$$= 0.694 + 0.631 + 0.473$$
$$I = 1.798\,A$$

Q. Find i_o using superposition theorem for the circuit shown in the figure.



Sol:

6V source acting alone



By inspection method,

$$(24K)\,I_1 + (-12K)\,I_2 = -6$$

$$(-12K)\,I_1 + (36K)\,I_2 = +6$$

$K \rightarrow 10^3$

$$I_1 = -2\times10^{-4}\,A$$

$$I_2 = +1\times10^{-4}\,A$$

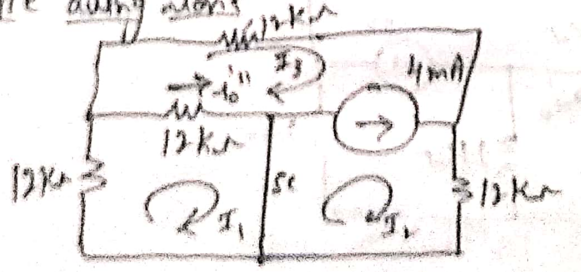
$$i_o' = I_1 - I_2$$

$$i_o' = -2\times10^{-4} - (+1\times10^{-4})$$

$$i_o' = -3\times10^{-4}\,A = -0.3\times10^{-3}$$

$$i_o' = -0.3\,mA$$

4mA source acting alone



KCL equation for supermesh 2&3, $I_2 - I_3 = 4\times10^{-3}$

KVL equation for supermesh 2&3, $12KI_3 + 12KI_2 + 12K(I_3 - I_1) = 0$

KVL equation for essential mesh 1 : $12KI_1 + 12K(I_1 - I_3) = 0$

Sorting the above equations,

$$(0)I_1 + (1)I_2 + (-1)I_3 = 4\times10^{-3}$$

$$(-12K)I_1 + (12K)I_2 + (24K)I_3 = 0$$

$$(24K)I_1 + (0)I_2 + (-12K)I_3 = 0$$

$$I_1 = -8\times10^{-4}A$$

$$I_2 = 2.4\times10^{-3}A$$

$$I_3 = -1.6\times10^{-3}A$$

$$i_o'' = I_1 - I_3$$

$$= -8\times10^{-4} - (-1.6\times10^{-3})$$

$$= 8\times10^{-4}A = 0.8\times10^{-3}A$$

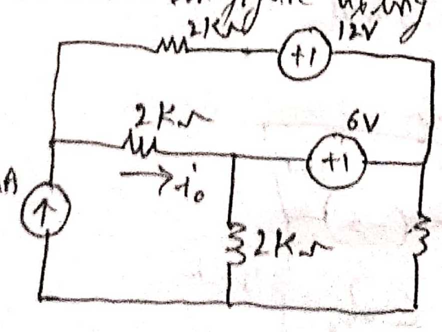
$$i_o'' = 0.8\ mA$$

By superposition theorem $i_o = i_o' + i_o''$

$$= (-0.3 + 0.8)\ mA$$

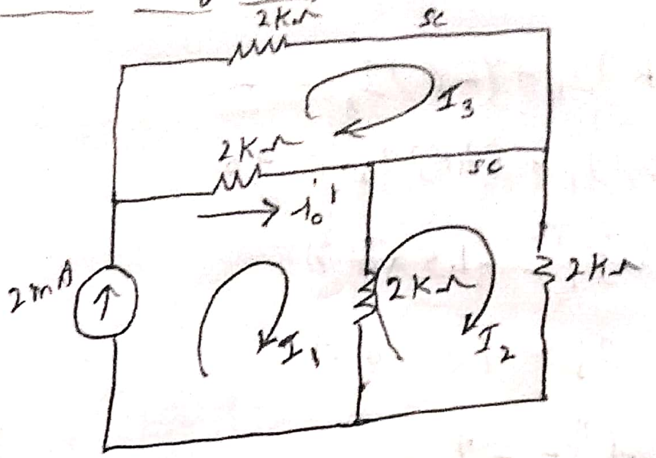
$$i_o = 0.5\ mA$$

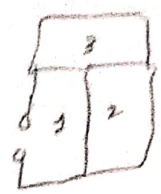
Q. Find i_o for the circuit shown in figure using superposition theorem



Sol:

2 mA source acting alone,





KCL equation for non-essential mesh 1 : $I_1 = +2\times10^{-3}$

KVL equation for essential mesh 2 : $2K(I_2 - I_1) + 2K I_2 = 0$

KVL equation for essential mesh 3 : $2K I_3 + 2K(I_3 - I_1) = 0$

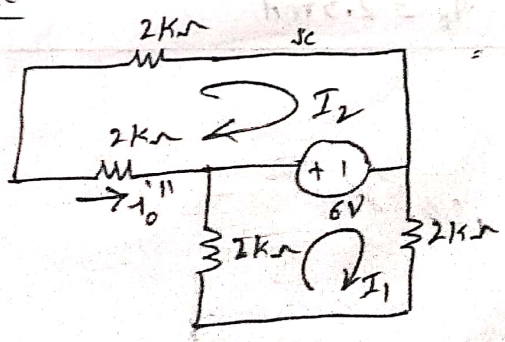
Sorting the above equations,

$$(1)I_1 + (0)I_2 + (0)I_3 = 2\times10^{-3}$$
$$(-2K)I_1 + (4K)I_2 + (0)I_3 = 0$$
$$(-2K)I_1 + (0)I_2 + (4K)I_3 = 0$$

$$I_1 = 2\times10^{-3}\ A$$
$$I_2 = 1\times10^{-3}\ A$$
$$I_3 = 1\times10^{-3}\ A$$

$$i_0' = I_1 - I_3$$
$$= 2\times10^{-3} - 1\times10^{-3}$$
$$= 1\times10^{-3}$$
$$i_0' = 1\,mA$$

6V source acting alone



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By inspection method,

$$(4K)I_1 + (-0)I_2 = -6$$

$$(-0)I_1 + (4K)I_2 = +6$$

$$I_1 = -1.5\times10^{-3}A$$

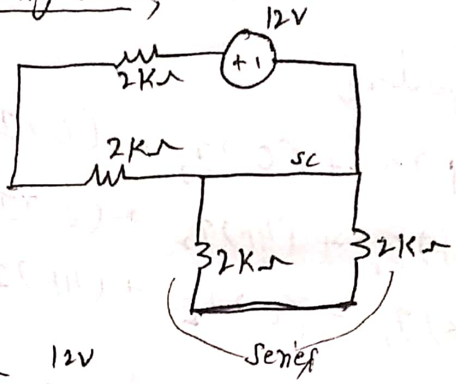
$$I_2 = 1.5\times10^{-3}A$$

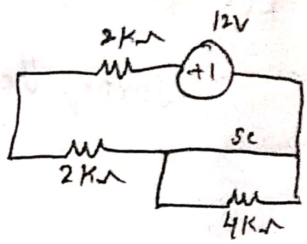
$$i_0'' = -I_2$$

$$i_0'' = -1.5\times10^{-3}A$$

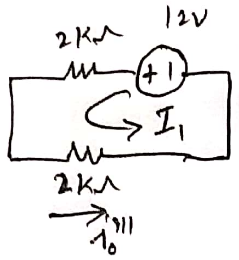
$$i_0'' = -1.5mA$$

12V source acting alone,





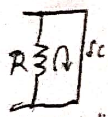
4KΩ is across short circuit. Remove 4KΩ (No current will flow through 4KΩ)



$$I_1 = \frac{12}{4K} = 3mA$$

$$i_0''' = I_1 = 3mA$$

In general,



In a closed path if there is one resistance only, then remove that resistance.

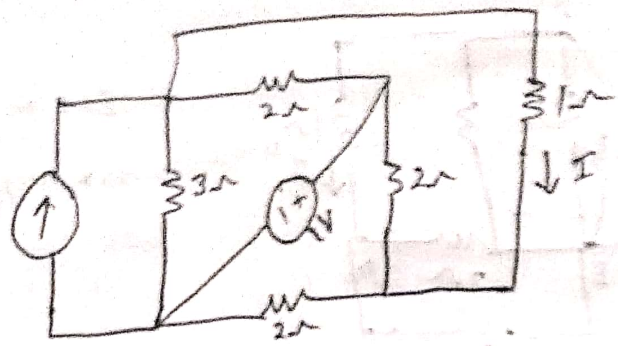
By superposition theorem,

$$i_0 = i_0' + i_0'' + i_0'''$$

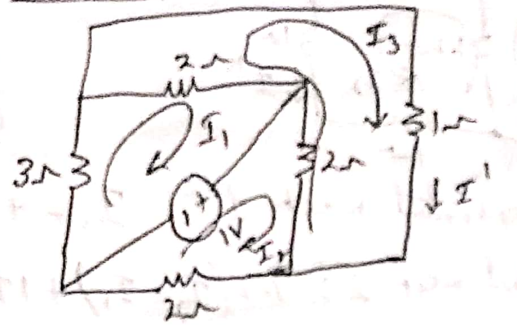
$$i_0 = (1 - 1.5 + 3)mA$$

$$i_0 = 2.5mA$$

6. Find the current in 1Ω resistor using superposition theorem for the circuit shown in figure.



Sol: IV source acting alone,



By inspection method,

$$(5)I_1 + (-0)I_2 + (-2)I_3 = -1$$

$$(-0)I_1 + (4)I_2 + (-2)I_3 = +1$$

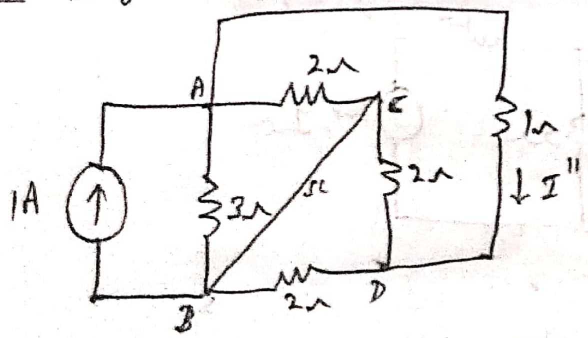
$$(-2)I_1 + (-2)I_2 + (5)I_3 = 0$$

$$I_1 = -0.1875\,A$$

$$I_2 = 0.2656\,A$$

$$I' = I_3 = 0.0312\,A$$

1A source acting alone,



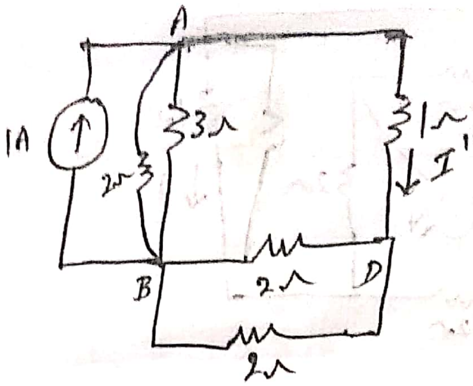
B&C are the same nodes

2Ω is between A & B (C)

2Ω is between B & D (C)

3Ω is between A & B (C)

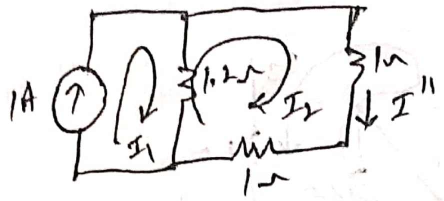
Rewriting the circuit,
(Retaining A, B & D)

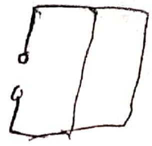


3 & 2 are parallel ⇒ 3Ω (3+1)

2 & 2 are parallel ⇒ 1Ω

Now the circuit is reduced from 4 meshes to 2 meshes





KCL equation to non essential mesh 1: $I_1 = +1$

KVL equation to essential mesh 2: $1.2(I_2 - I_1) + 1I_2 + 1I_2 = 0$

$$(1) I_1 + (0) I_2 = 1$$

$$(-1.2) I_1 + (3.2) I_2 = 0$$

$$I_1 = 1A$$

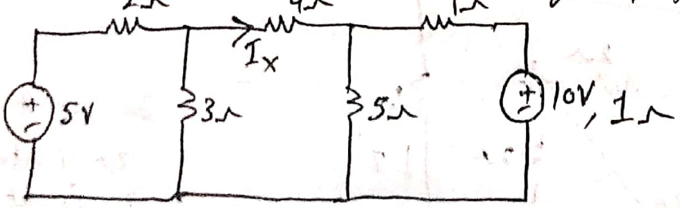
$$I'' = I_2 = 0.375$$

By superposition theorem, $I = I' + I''$

$$= 0.0312 + 0.375$$

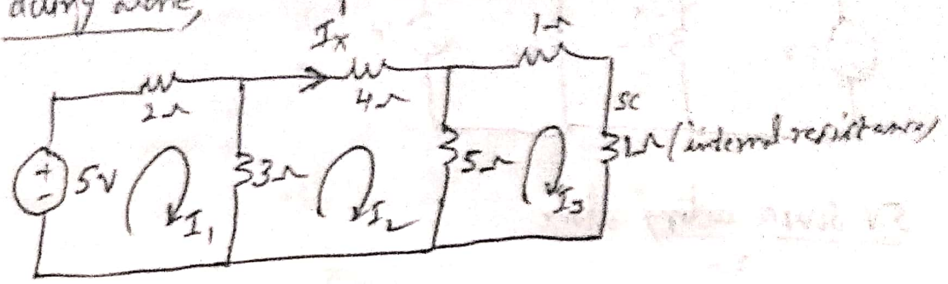
$$I = 0.4062A$$

Q. For the network shown in figure find I_x using superposition theorem.



Sol: Internal resistance of 10V is 1Ω, that should be considered.

5V source acting alone,

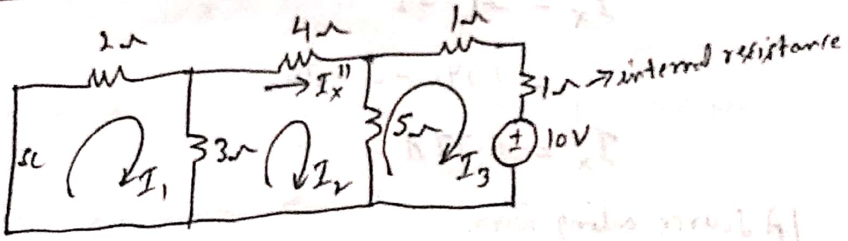


By inspection method,

$$(5)I_1 + (-3)I_2 + (-0)I_3 = 5$$
$$(-3)I_1 + (12)I_2 + (-5)I_3 = 0$$
$$(-0)I_1 + (-5)I_2 + (7)I_3 = 0$$

$$I_1 = 1.271A$$
$$I_x' = I_2 = 0.452A$$
$$I_3 = 0.323A$$

10V source acting alone



By inspection method,

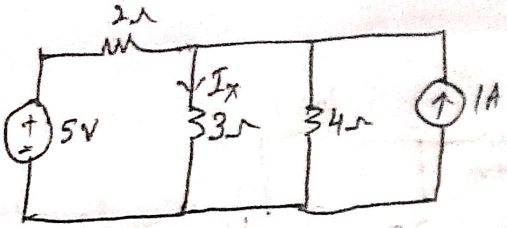
$$(5)I_1 + (-3)I_2 + (0)I_3 = 0$$
$$(-3)I_1 + (12)I_2 + (-5)I_3 = 0$$
$$(0)I_1 + (-5)I_2 + (7)I_3 = -10$$

$$I_1 = -0.646A$$
$$I_x'' = I_2 = -1.077A$$
$$I_3 = -2.198A$$

By superposition theorem, $I_x = I_x' + I_x''$

$$= 0.452 - 1.077$$
$$I_x = -0.625A$$

Q. For the Circuit shown in figure, find I_x using superposition theorem.



Sol:

5V Source acting alone,

By inspection method,

$$(5)I_1 + (-3)I_2 = 5$$
$$(-3)I_1 + (7)I_2 = 0$$

$$I_1 = 1.346A$$
$$I_2 = 0.576A$$

$$I_x' = I_1 - I_2$$
$$= 1.346 - 0.576$$
$$I_x' = 0.77A$$

1A Source acting alone,

KCL equation for non-essential mesh 3: $I_3 = -1$

KVL equation for essential mesh 1: $2I_1 + 3(I_1 - I_2) = 0$

KVL equation for essential mesh 2: $3(I_2 - I_1) + 4(I_2 - I_3) = 0$

Sorting the equations.

$$(0)I_1 + (0)I_2 + (1)I_3 = -1$$
$$(5)I_1 + (-3)I_2 + (0)I_3 = 0$$
$$(-3)I_1 + (7)I_2 + (-4)I_3 = 0$$

$$I_1 = -0.461\,A$$

$$I_2 = -0.769\,A$$

$$I_3 = -1\,A$$

$$I_x'' = I_1 - I_2$$

$$= -0.461 - (-0.769)$$

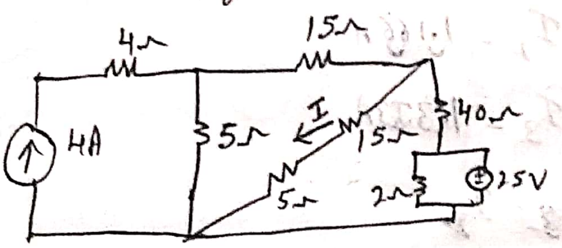
$$I_x'' = 0.308\,A$$

By superposition theorem, $I_x = I_x' + I_x''$

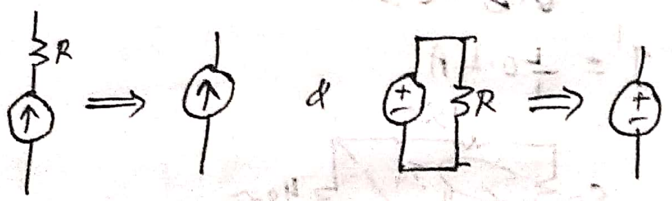
$$= 0.77 + 0.308$$

$$I_x = 1.078\,A$$

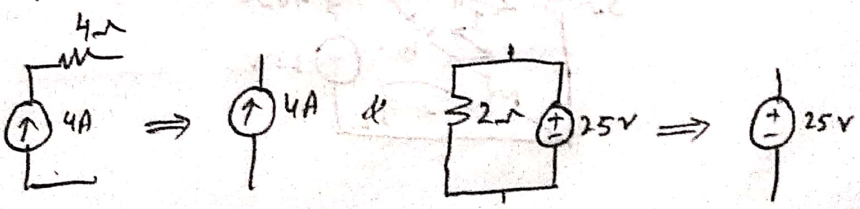
Q. For the Circuit shown in figure, find I using superposition theorem.



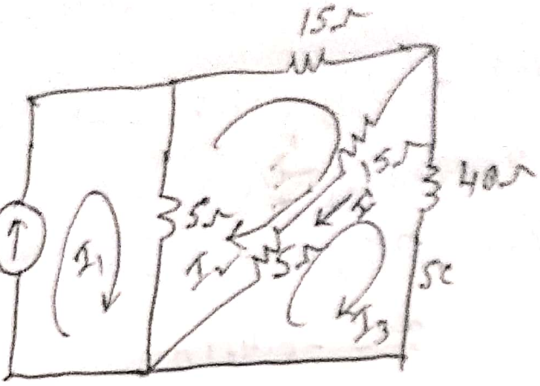
Sol: **Note:**



So



Source 4A acting alone



KCL equation for non-essential mesh 1 : $I_1 = +4$

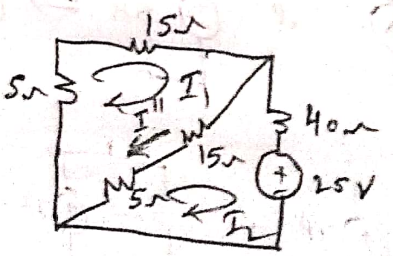
KVL equation for essential mesh 2 : $5(I_2 - I_1) + 15I_2 + 15(I_2 - I_3)$
$+ 5(I_2 - I_3) = 0$

KVL equation for essential mesh 3 : $5(I_3 - I_2) + 15(I_3 - I_2)$
$+ 40 I_3 = 0$

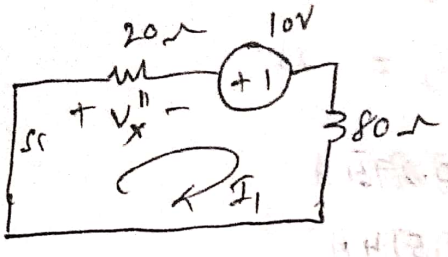
$$(1) I_1 + (0) I_2 + (0) I_3 = 4$$
$$(-5) I_1 + (40) I_2 + (-20) I_3 = 0$$
$$(0) I_1 + (-20) I_2 + (60) I_3 = 0$$

$$I_1 = 4A$$
$$I_2 = 0.6 A$$
$$I_3 = 0.2 A$$
$$I' = I_2 - I_3$$
$$= 0.6 - 0.2$$

25 V Source acting alone, $I' = +0.4 A$



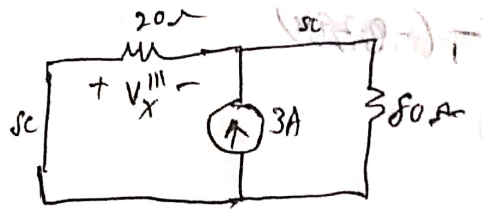
10 V source acting alone,



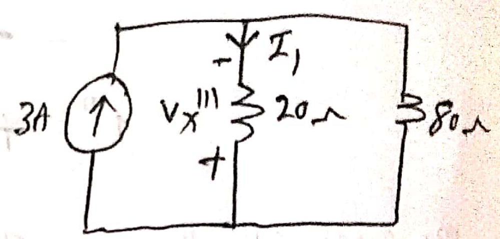
$$I_1 = \frac{-10}{(20+80)} = -0.1A$$

$$V_x'' = 20 I_1 = 20 \times -0.1 = -2V$$

3A source acting alone



⇒

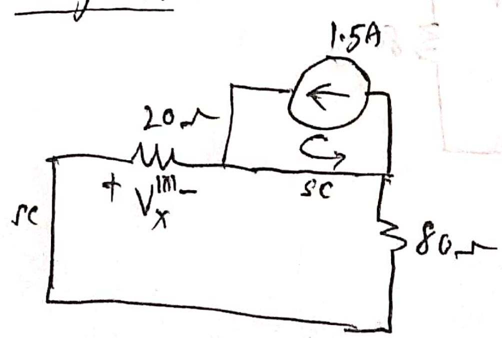


By current division method

$$I_1 = \frac{3 \times 80}{(20+80)} = 2.4A$$

$$V_x''' = \underset{\substack{\downarrow \\ \text{Current is} \\ \text{entering '−' terminal}}}{-} 20 I_1 = -20 \times 2.4 = -48V$$

1.5A source acting alone



No current flows through 20Ω & 80Ω

$$V_x'''' = 0V$$

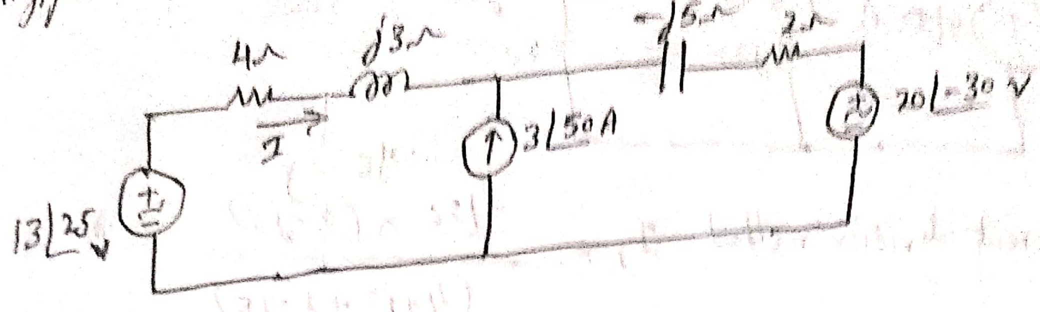
By superposition theorem,

$$V_x = V_x' + V_x'' + V_x''' + V_x''''$$

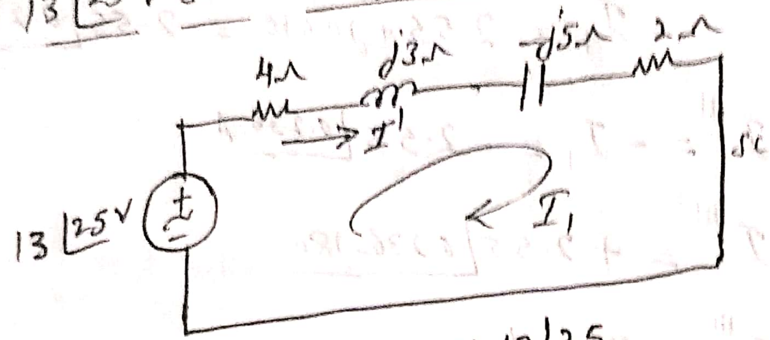
$$V_x = (3.2) + (-2) + (-48) + 0$$

$$V_x = -46.8V$$

Q. By using superposition principle, find the current through $(4+j3)\,\Omega$ as shown in figure.

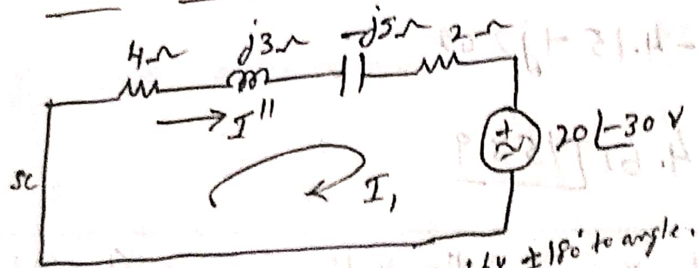


Sol: $13\angle 25$ V source acting alone



$$I' = I_1 = \frac{13\angle 25}{(4+j3-j5+2)} = 1.492+j1.413 = 2.05\angle 43.43 \text{ A}$$

$20\angle -30$ V source acting alone

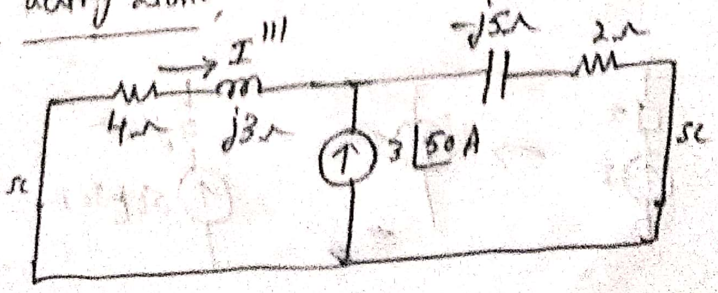


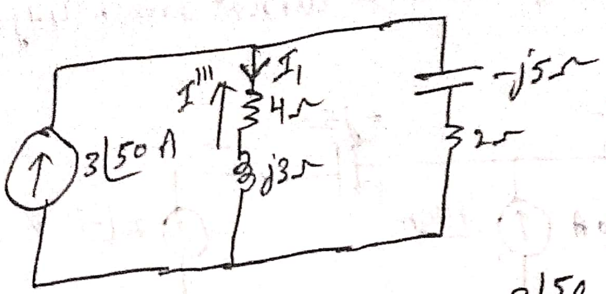
Remove '−' by ±180° to angle. If not calculator will show 'Math error'

$$I'' = I_1 = \frac{-20\angle -30}{(4+j3-j5+2)} = \frac{+20\angle -30+180}{(6-j2)}$$

$$I'' = \frac{20\angle 150}{(6-j2)} = -3.098+j0.633 = 3.16\angle 168.43 \text{ A}$$

$3\angle 50$ A source acting alone,





By current division method, $I_1 = \dfrac{3\angle 50 \times (2-j5)}{(4+j3+2-j5)}$

$$I_1 = 2.55 + j0.010 = 2.55\angle 0.236 \text{ A}$$

$$I''' = -I_1 = -2.55\angle 0.236 \text{ A}$$

$$I''' = +2.55\angle 0.236-180$$

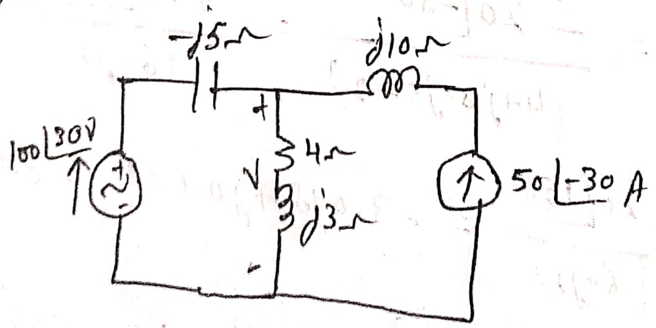
$$I''' = 2.55\angle -179.7 \text{ A}$$

$$I = I' + I'' + I''' = 2.05\angle 43.43 + 3.16\angle 168.43 + 2.55\angle -179.7$$

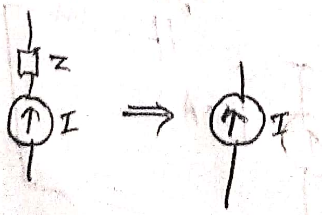
$$I = -4.15 + j2.02$$

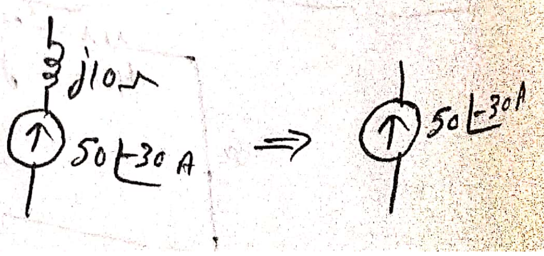
$$I = 4.62\angle 153.9$$

Q. Using superposition theorem, find the voltage across $(4+j3)\Omega$ in the circuit shown in figure.

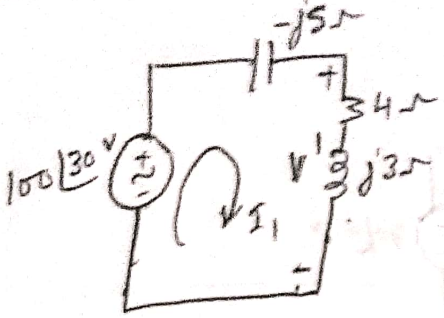


Sol:





Sol: $100\angle 30$ V source acting alone



$$I_1 = \frac{100\angle 30}{(-j5+4+j3)} = \frac{100\angle 30}{(4-j2)}$$

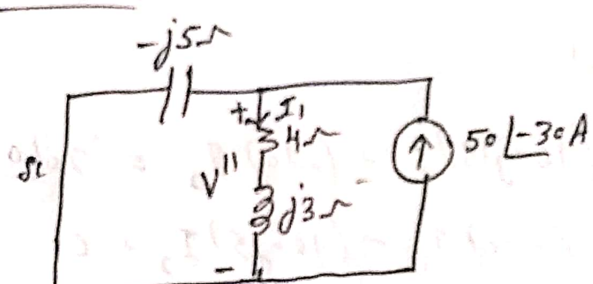
$$I_1 = 12.32 + j18.66 = 22.3\angle 56.56 \text{ A}$$

$$V' = (4+j3) \times I_1$$

$$V' = (4+j3) \times 22.3\angle 56.56$$

$$V' = -6.67 + j111.3 = 111.5\angle 93.42 \text{ V}$$

$50\angle -30$ A acting alone



By current division method, $I_1 = \dfrac{50\angle -30 \times -j5}{(4+j3-j5)}$

$$I_1 = -33.49 - j55.8 = 55.9\angle -93.4$$

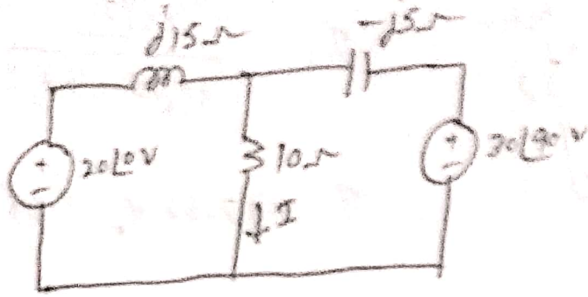
$$V'' = (4+j3) \times I_1 = (4+j3) \times 55.9\angle -93.4$$

$$V'' = 154.14 - j233.15 = 279.5\angle -56.53$$

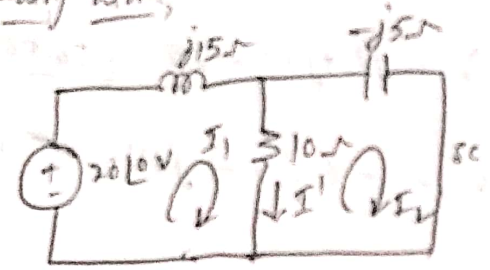
By superposition theorem, $V = V' + V'' = 111.5\angle 93.42 + 279.5\angle -56.53$

$$V = 147.4 - j121.8 = 191.3\angle -39.56 \text{ V}$$

Q. Find the current I by using superposition theorem for the circuit shown in the figure.



Sol: $20\angle 0°$ V acting alone,



By inspection method,

$$(10+j15)I_1 + (-10)I_2 = 20\angle 0$$
$$(-10)I_1 + (10-j5)I_2 = 0$$

By Cramer's rule,

$$I_1 = \frac{\begin{vmatrix} 20\angle 0 & -10 \\ 0 & (10-j5) \end{vmatrix}}{\begin{vmatrix} (10+j15) & -10 \\ -10 & (10-j5) \end{vmatrix}} = \frac{20\times(10-j5)}{(10+j15)\times(10-j5)-10\times 10}$$

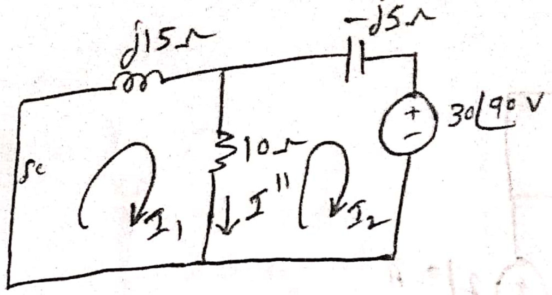
$$I_1 = \frac{(200-j100)}{(75+j100)} = 0.32-j1.76 = 1.788\angle -79.69$$

$$I_2 = \frac{\begin{vmatrix} (10+j15) & 20\angle 0° \\ -10 & 0 \end{vmatrix}}{(75+j100)} = \frac{200\angle 0}{(75+j100)} = 0.96-j1.28 = 1.6\angle -53.13$$

$$I' = I_1 - I_2 = 1.788\angle -76.69 - 1.6\angle -53.13$$

$$I' = -0.548 - j0.459 = 0.715\angle -140 \text{ A}$$

$30\angle 90$ V acting alone,



By inspection method,

$$(10+j15)I_1 + (-10)I_2 = 0$$
$$(-10)I_1 + (10-j5)I_2 = -30\angle 90$$

By Cramer's rule,

$$I_1 = \frac{\begin{vmatrix} 0 & -10 \\ -30\angle 90 & (10-j5) \end{vmatrix}}{(75+j100)} = \frac{-30\angle 90 \times 10}{(75+j100)}$$

↓ No change in denominator

$$I_1 = \frac{+30\angle 90-180 \times 10}{(75+j100)} = \frac{30\angle -90 \times 10}{(75+j100)}$$

$$I_1 = -1.92 - j1.44 = 2.4\angle -143.13$$

$$I_2 = \frac{\begin{vmatrix} (10+j15) & 0 \\ -10 & -30\angle 90 \end{vmatrix}}{(75+j100)} = \frac{(10+j15)\times -30\angle 90}{(75+j100)}$$

$$I_2 = \frac{(10+j15)\times 30\angle 90-180}{(75+j100)} = \frac{(10+j15)\times 30\angle -90}{(75+j100)}$$

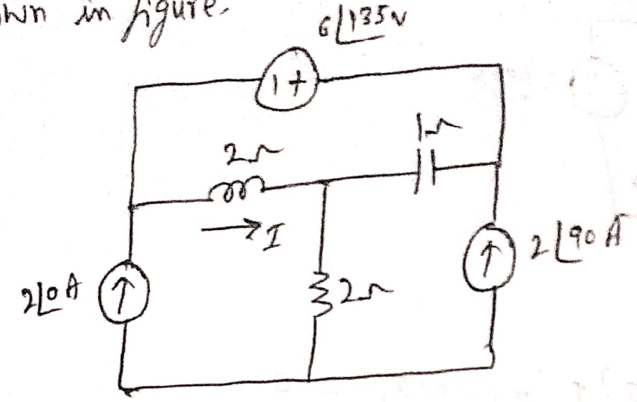
$$I_2 = 0.24 - j4.32 = 4.32\angle -86.82 \text{ A}$$

$$I'' = I_1 - I_2 = 2.4\angle -143.13 - 4.32\angle -86.82 = -2.15 + j2.87$$
$$= 3.59\angle 126.9 \text{ A}$$

$$I = I' + I'' = 0.715\angle -140 + 3.59\angle 126.9$$

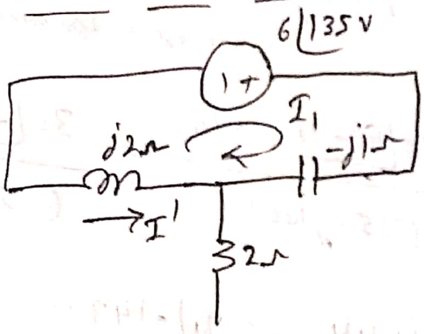
$$I = -2.7 + j2.41 = 3.62\angle 138.26$$

Q. Using superposition theorem, obtain the response I for the network shown in figure.



Sol: +j should be attached with $X_L \Rightarrow +j2\Omega$
$-j$ should be attached with $X_C \Rightarrow -j1\Omega$

6∠135 V source acting alone



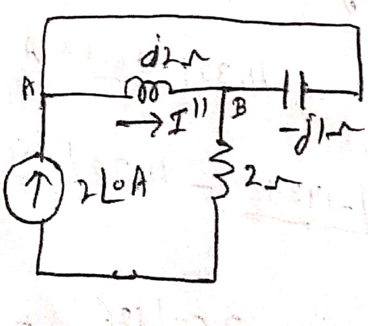
(No current flow through 2Ω)

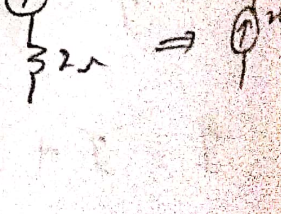
$$I_1 = \frac{6\angle 135}{(j2-j1)} = 4.24 + j4.24 = 6\angle 45 \text{ A}$$

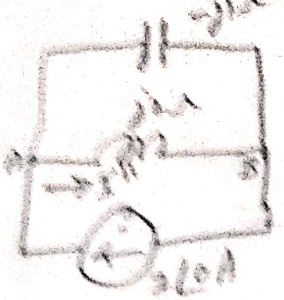
$$I' = -I_1 = -6\angle 45 = +6\angle 45-180$$

$$I' = 6\angle 135 \text{ A}$$

2∠0 A source acting alone

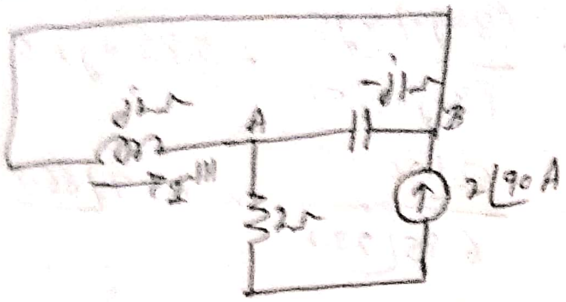


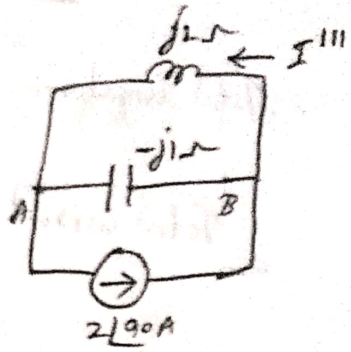




$$I'' = \frac{2\angle 0 \times -j1}{(-j1+j2)} = 2\angle 180\ A$$

$2\angle 90\ A$ source acting alone,





$$I''' = \frac{2\angle 90^\circ \times -j1}{(j2-j1)} = 2\angle -90\ A$$

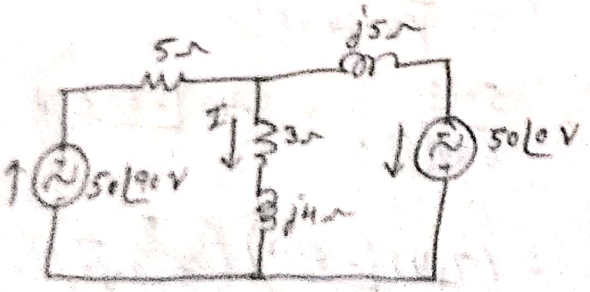
$$I = I' + I'' + I'''$$

$$I = 6\angle -135 + 2\angle 180 + 2\angle -90$$

$$I = 8.828\angle -135\ A$$

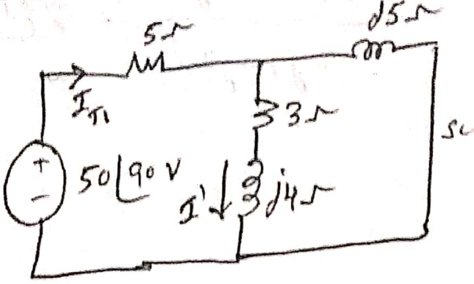
---

6. Apply the superposition theorem to the network shown in figure and obtain the current in the $(3+j4)\Omega$ impedance.



Sol:

$50\angle 90\,V$ acting alone,



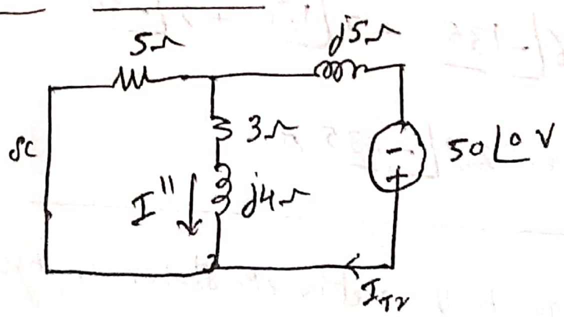
Total impedance $Z_{T1} = 5 + \dfrac{(3+j4) \times j5}{(3+j4+j5)} = 5.83 + j2.5 = 6.35\angle 23.2\ \Omega$

Total current $I_{T1} = \dfrac{50\angle 90}{6.35\angle 23.2} = 7.87\angle 66.8\ A$

By current division method, $I' = I_{T1}\left(\dfrac{j5}{3+j4+j5}\right) = 7.87\angle 66.8\left(\dfrac{j5}{3+j9}\right)$

$I' = 4.15\angle 85.3\ A$

$50\angle 0\,V$ acting alone,



Total impedance, $Z_{T2} = j5 + \dfrac{(3+j4)\times 5}{(3+j4+5)} = 2.5 + j6.25 = 6.74\angle 68.2\ \Omega$

Total current, $I_{T2} = \dfrac{50\angle 0}{6.74\angle 68.2} = 7.42\angle -68.2\ A$

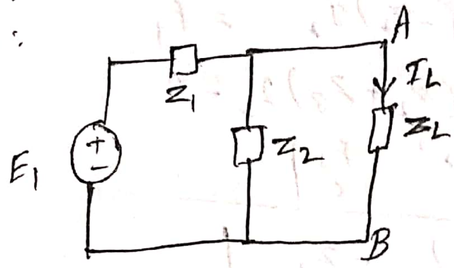
$I'' = -(7.42\angle -68.2)\left(\dfrac{5}{5+3+j4}\right) = 4.15\angle 85.3\ A$

$I = I' + I'' = 4.15\angle 85.3 + 4.15\angle 85.3 = 8.30\angle 85.3\ A$

Q. State, explain and prove Thevenin's theorem.

Sol: **Statement**: A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{th} in series with a resistance R_{th} (or impedance Z_{th}), where V_{th} is the open-circuit voltage at the terminals and R_{th} is the equivalent resistance (or Z_{th} - equivalent impedance) at the terminals when the independent sources are turned off or R_{th} (Z_{th}) is the ratio of open circuit voltage to the short circuit current at the terminal pair.

Explanation:



fig(a)

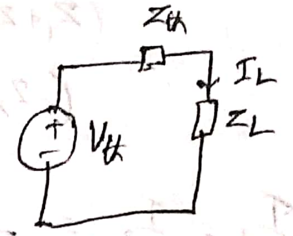
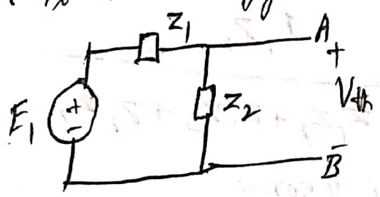


fig (b)

The Thevenin equivalent circuit for the circuit shown in fig (a) across A-B is shown in fig(b).

The V_{th} is the voltage obtained across terminals A-B when Z_L is removed. It is shown in fig(c). While obtaining V_{th}, any of the network simplification techniques may be used.



fig(c)

Z_{th} is the equivalent impedance as viewed through terminals A-B with Z_L removed and voltage sources replaced by short circuit and current source by open circuit. This is shown in fig (d).

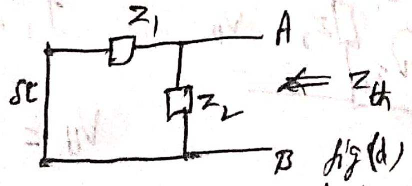
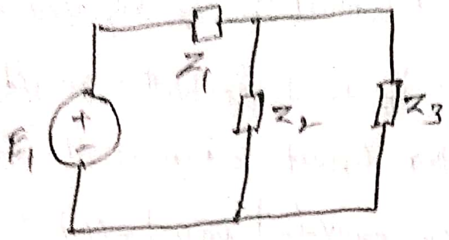


fig (d)

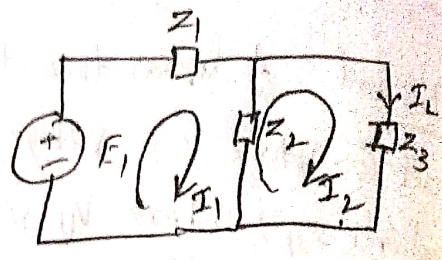
By the Thevenin equivalent circuit (fig (b))

$$I_L = \frac{V_{th}}{Z_L + Z_{th}}$$

Proof: Consider a network shown in fig(e). Let us obtain the current through the impedance Z_3 by mesh analysis first.



fig(e)



fig(f)

By inspection method,

$$(Z_1+Z_2)I_1 + (-Z_2)I_2 = E_1$$

$$(-Z_2)I_1 + (Z_2+Z_3)I_2 = 0$$

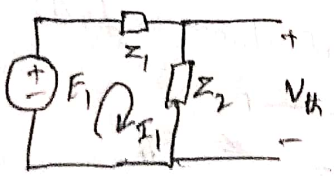
$$I_L = I_2 = \frac{\begin{vmatrix} (Z_1+Z_2) & E_1 \\ -Z_2 & 0 \end{vmatrix}}{\begin{vmatrix} (Z_1+Z_2) & -Z_2 \\ -Z_2 & (Z_2+Z_3) \end{vmatrix}}$$

$$I_L = I_2 = \frac{E_1 Z_2}{Z_1 Z_2 + Z_1 Z_3 + \cancel{Z_2^2} + Z_2 Z_3 - \cancel{Z_2^2}}$$

$$I_L = I_2 = \frac{E_1 Z_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} \quad -①$$

Now I_L is obtained by Thevenin's theorem.

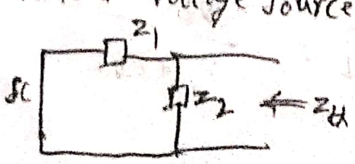
To find V_{th}: Remove the load impedance, i.e., Z_3



$$I_1 = \frac{E_1}{Z_1+Z_2}$$

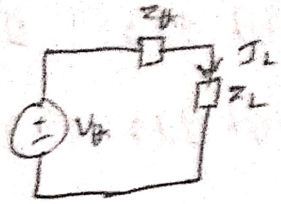
$$V_{th} = I_1 Z_2 = \frac{E_1 Z_2}{Z_1+Z_2}$$

To find Z_{th}: Remove the load impedance, i.e., Z_3, short circuit the voltage source.



$$Z_{th} = \frac{Z_1 Z_2}{Z_1+Z_2} \quad (Z_1 \,\&\, Z_2 \text{ are parallel})$$

Thevenin equivalent Circuit



$$I_L = \frac{V_{th}}{Z_L + Z_{th}} = \frac{V_{th}}{Z_3 + Z_{th}}$$

$$I_L = \frac{\dfrac{E_1 Z_2}{Z_1+Z_2}}{Z_3 + \dfrac{Z_1 Z_2}{Z_1+Z_2}} = \frac{\dfrac{E_1 Z_2}{Z_1+Z_2}}{\dfrac{Z_3 Z_1 + Z_3 Z_2 + Z_1 Z_2}{Z_1+Z_2}}$$

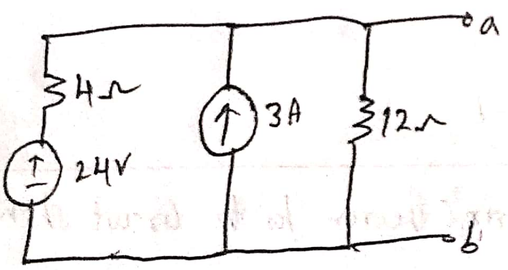
$$I_L = \frac{E_1 Z_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} \quad -(2)$$

Comparing Eq.(2) with Eq(1), Thevenin's theorem is verified (proved)

Q. What are the limitations of Thevenin's theorem.

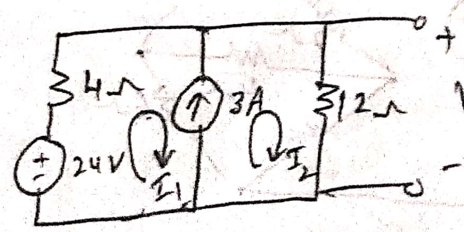
Sol: i) Not applicable to the circuits consisting of non-linear elements.
ii) Not applicable to unilateral networks.
iii) There should not be magnetic coupling between the load and circuit to be replaced by Thevenin's theorem.
iv) In the load side, there should not be controlled sources, controlled from some other part of the circuit.

Q. Find the Thevenin equivalent circuit across terminals a, b for the network shown in figure.



Sol: Load resistance R_L is not given in problem.

To find V_{th} (or V_{oc}): open circuit voltage



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KCL equation for super mesh 1 & 2 : $I_2 - I_1 = 3$

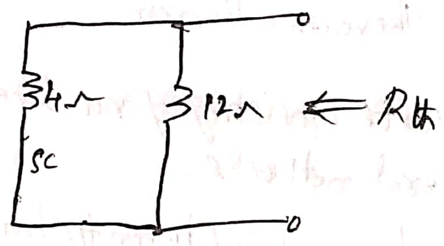
KVL equation for super mesh 1 & 2 : $-24 + 4I_1 + 12I_2 = 0$

$$(-1)I_1 + (1)I_2 = 3$$
$$(4)I_1 + (12)I_2 = 24$$

$$I_1 = -0.75\,A$$
$$I_2 = 2.25\,A$$

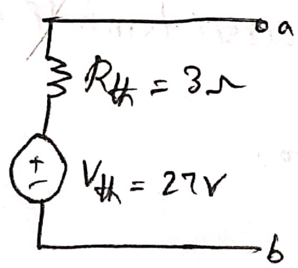
$$V_{th} = 12I_2 = 12 \times 2.25 = 27\,V$$

To find R_{th} : open circuit current source, short circuit voltage source,

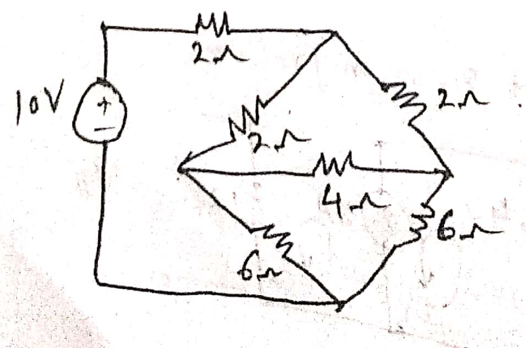


$$R_{th} = \frac{4 \times 12}{(4+12)} = 3\,\Omega$$

Thevenin eq. ckt across a-b

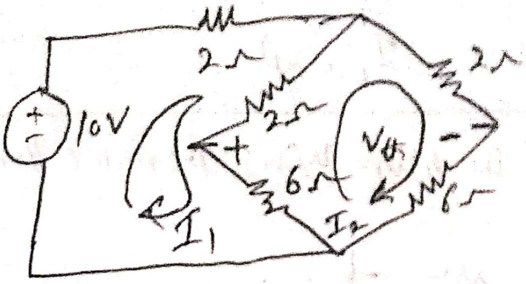


Q. Find the current in 4Ω using Thevenin's theorem for the circuit shown in figure.



Sol: $R_L = 4\Omega$ (given)

To find V_{th}: Remove R_L



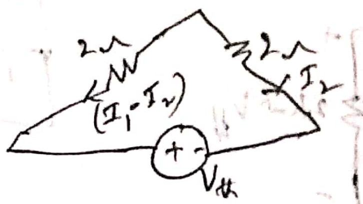
By inspection method,

$$(10)I_1 + (-8)I_2 = 10$$

$$(-8)I_1 + (16)I_2 = 0$$

$$I_1 = 1.666A$$

$$I_2 = 0.8333A$$



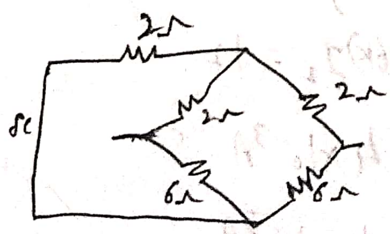
KVL: $2(I_1 - I_2) + V_{th} - 2I_2 = 0$

$$V_{th} = 2I_2 - 2(I_1 - I_2)$$

$$V_{th} = 2\times0.8333 - 2\times(1.666 - 0.8333)$$

$$V_{th} = 0V$$

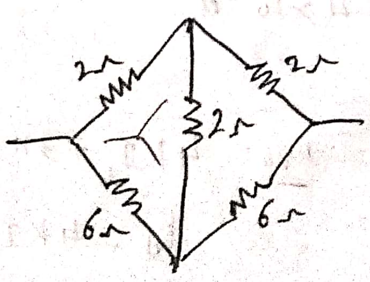
To find R_{th}:



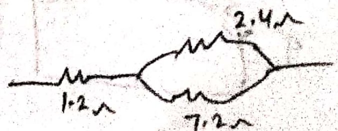
$\Delta \to Y \Rightarrow \frac{6\times2}{(2+6+2)} = 1.2\Omega$

$\frac{2\times2}{(2+6+2)} = 0.4\Omega$

$\frac{6\times2}{(2+6+2)} = 1.2\Omega$

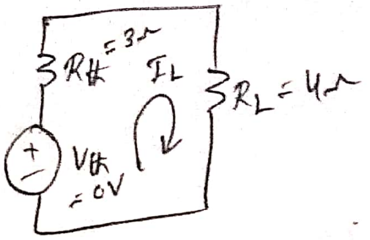


$\Delta - Y$ transformation



$$R_{th} = 1.2 + \frac{2.4\times7.2}{(2.4+7.2)} = 3\Omega$$

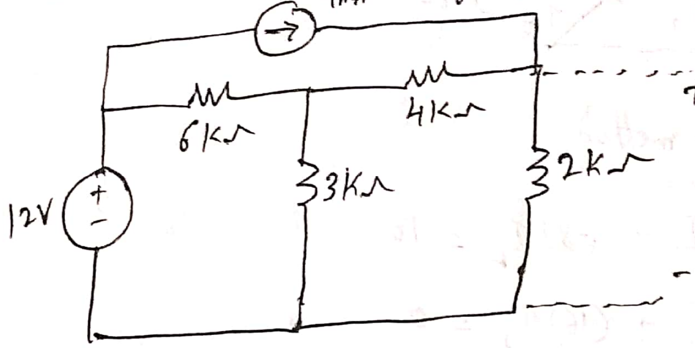
Thevenin eq. ckt



$$I_L = \frac{V_{th}}{(R_L + R_{th})} = \frac{0}{(4+3)}$$

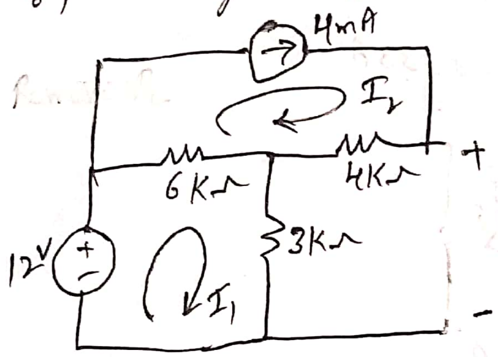
$$\therefore I_L = 0A$$

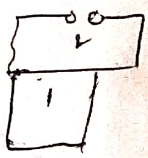
Q. Find V_o in the circuit shown in the figure using Thevenin's theorem.



Sol: We need V_o, the voltage across $2K\Omega$, so $R_L = 2K\Omega$

To find V_{th}:
(Remove R_L)



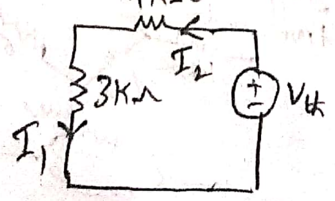


KCL equation for non-essential mesh 2: $I_2 = +4\times10^{-3}$

KVL equation for essential mesh 1: $-12 + 6K(I_1 - I_2) + 3KI_1 = 0$

$$(0)I_1 + (1)I_2 = 4\times10^{-3}$$
$$(9K)I_1 + (-6K)I_2 = 12$$

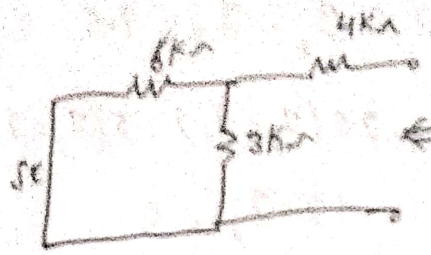
$$I_1 = 4\times10^{-3}A$$
$$I_2 = 4\times10^{-3}A$$



KVL: $4KI_2 + 3KI_1 - V_{th} = 0$

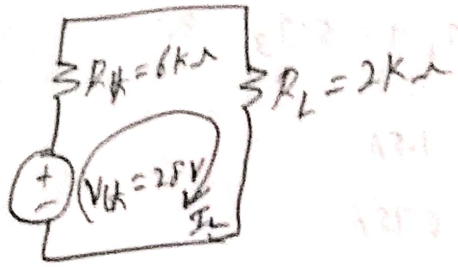
$$V_{th} = 4KI_2 + 3KI_1$$
$$V_{th} = 4\times10^3 \times 4\times10^{-3} + 3\times10^3 \times 4\times10^{-3}$$
$$V_{th} = 16 + 12 = 28V$$

To find R_{th}:



$$R_{th} = \frac{6\times3}{(6+3)} + 4 = 2+4 = 6K\Omega$$

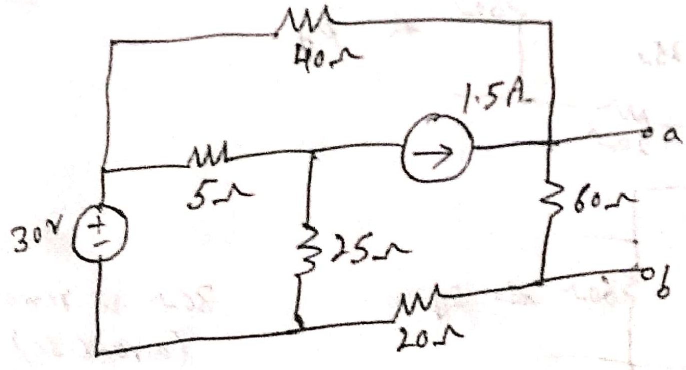
Thevenin equivalent circuit:



$$I_L = \frac{V_{th}}{(R_L+R_{th})} = \frac{28}{(6K+2K)} = 3.5\,mA$$

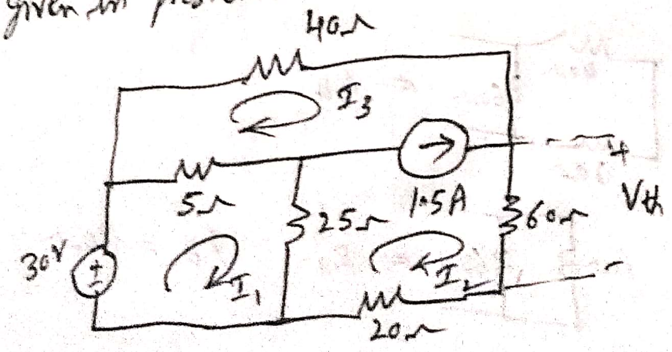
$$V_0 = 2K \times I_L = 2\times10^3 \times 3.5\times10^{-3} = 7V$$

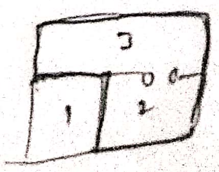
Q. Find Thevenin equivalent circuit for the circuit shown in figure across a-b



Sol: R_L is not given in problem.

To find V_{th}:





KCL equation for supermesh 2&3 : $I_2 - I_3 = 1.5$

KVL equation for supermesh 2&3 : $25(I_2 - I_1) + 5(I_3 - I_1) + 40 I_3 + 60 I_2$
$+ 20 I_2 = 0$

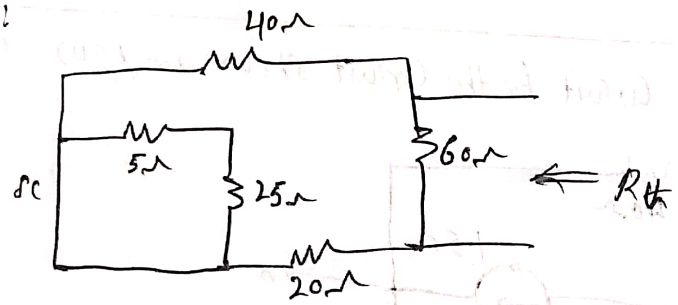
KVL equation for essential mesh 1 : $-30 + 5(I_1 - I_3) + 25(I_1 - I_2) = 0$

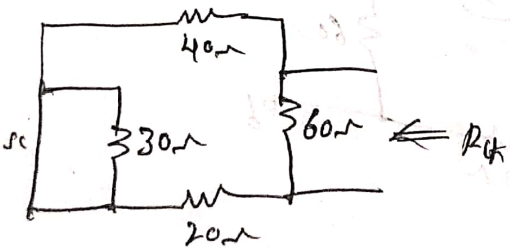
$$(0)I_1 + (1)I_2 + (-1)I_3 = 1.5$$
$$(-30)I_1 + (105)I_2 + (45)I_3 = 0$$
$$(30)I_1 + (-25)I_2 + (-5)I_3 = 30$$

$$I_1 = 1.5A$$
$$I_2 = 0.75A$$
$$I_3 = -0.75A$$

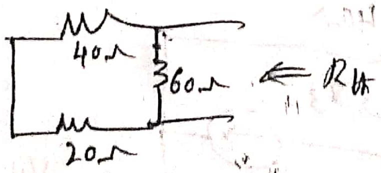
$$V_{th} = 60 I_2 = 60 \times 0.75$$
$$V_{th} = 45V$$

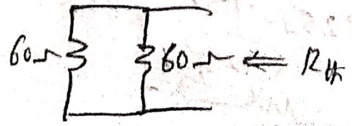
To find R_{th} :





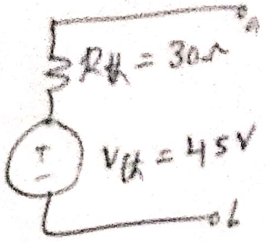
30Ω is removed (across sc)



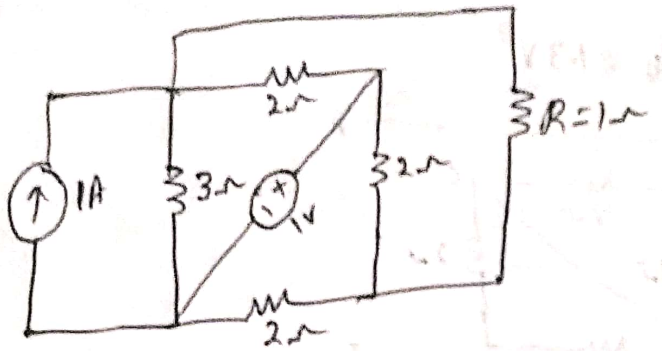


$R_{th} = 30Ω$

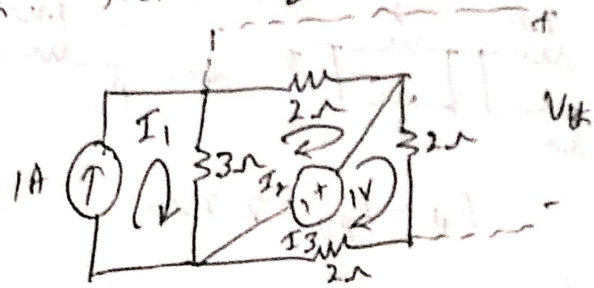
Thevenin equivalent ckt :

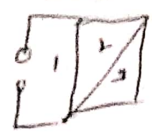


Q. Determine the current in $R = 1\Omega$ resistor for the network shown in figure using Thevenin's theorem.



Sol : To find V_{th} : Remove $R_L = 1\Omega$





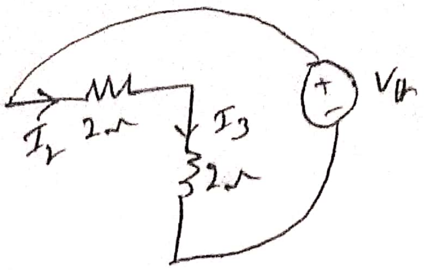
KCL for non-essential mesh 1 ; $I_1 = 1$

KVL for essential mesh 2 ; $3(I_2 - I_1) + 2I_2 + 1 = 0$

KVL for essential mesh 3 : $-1 + 2I_3 + 2I_3 = 0$

$$(1) I_1 + (0) I_2 + (0) I_3 = 1$$
$$(-3) I_1 + (5) I_2 + (0) I_3 = -1$$
$$(0) I_1 + (0) I_2 + (4) I_3 = +1$$

$$I_1 = 1A$$
$$I_2 = 0.4A$$
$$I_3 = 0.25A$$



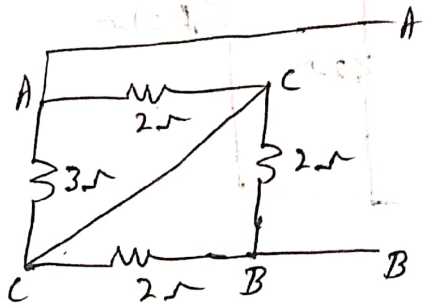
$$2I_2 + 2I_3 - V_{th} = 0$$

$$V_{th} = 2I_2 + 2I_3$$

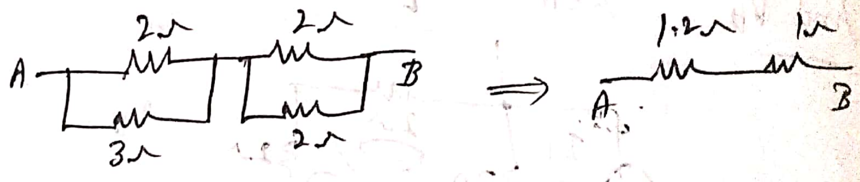
$$V_{th} = 2\times0.4 + 2\times0.25$$

$$V_{th} = 1.3\,V$$

To find R_{th} :

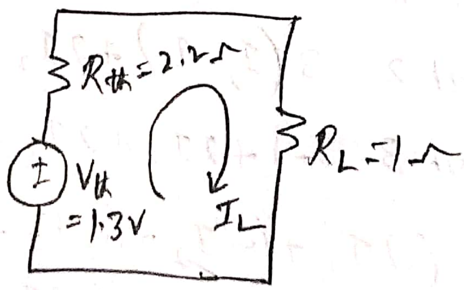


Re-writing the circuit



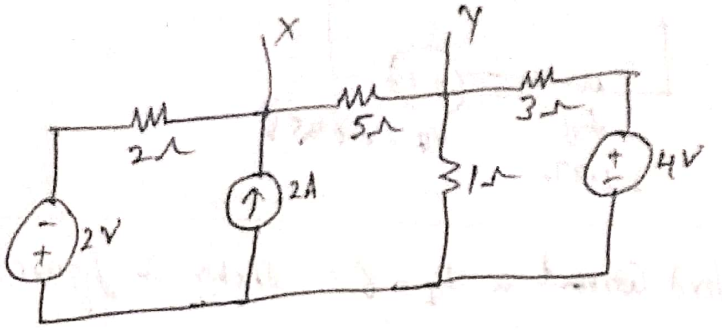
$$R_{th} = R_{AB} = 2.2\,\Omega$$

Thevenin Eq. ckt

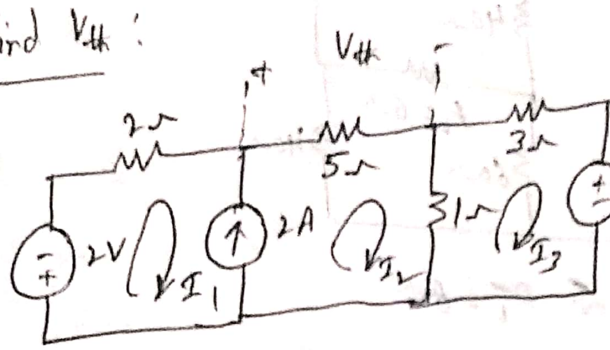


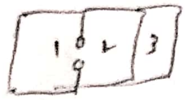
$$I_L = \frac{V_{th}}{R_L + R_{th}} = \frac{1.3}{(1+2.2)} = 0.4062A$$

Q. For the circuit shown in figure, find the Thevenin's equivalent circuit across X-Y terminals.



Sol: <u>To find V_{th}</u> :





KCL for Supermesh 1 & 2 : $I_2 - I_1 = 2$

KVL for Supermesh 1 & 2 : $+2 + 2I_1 + 5I_2 + 1(I_2 - I_3) = 0$

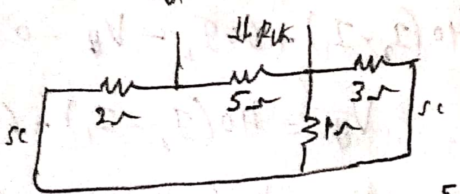
KVL for essential mesh 3 : $1\cdot(I_3 - I_2) + 3I_3 + 4 = 0$

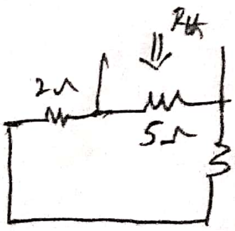
$$(-1)I_1 + (1)I_2 + (0)I_3 = 2$$
$$(2)I_1 + (6)I_2 + (-1)I_3 = -2$$
$$(0)I_1 + (-1)I_2 + (4)I_3 = -4$$

$$I_1 = -1.87\,A$$
$$I_2 = 0.129\,A$$
$$I_3 = -0.967\,A$$

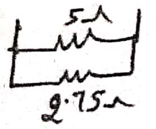
$$V_{th} = 5I_2 = 5 \times 0.129$$
$$V_{th} = 0.645\,V$$

<u>To find R_{th}</u> :



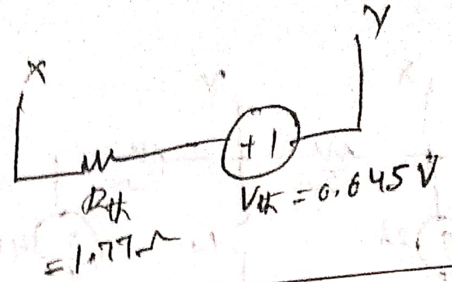


2Ω & 0.75Ω in series ⇒

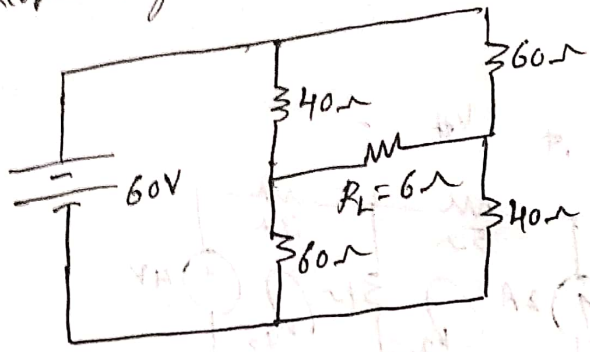


$$R_{th} = \frac{5 \times 2.75}{(5 + 2.75)} = 1.77\,\Omega$$

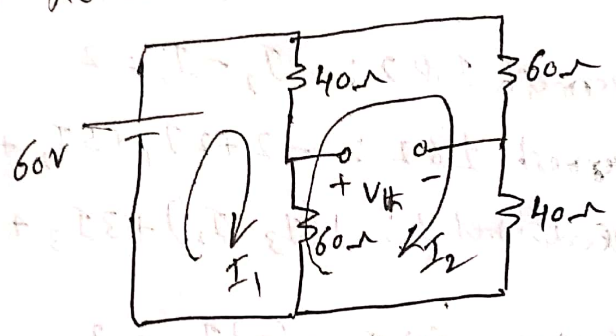
Thevenin Eq. ckt :



Q. Use Thevenin's theorem to find current in $R_L = 6\Omega$ shown in figure.

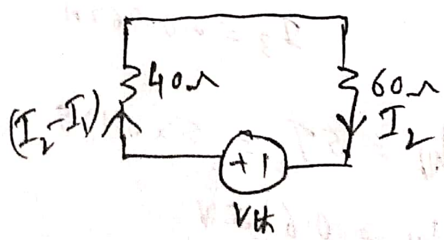


Sol: To find V_{th} : Remove $R_L = 6\Omega$



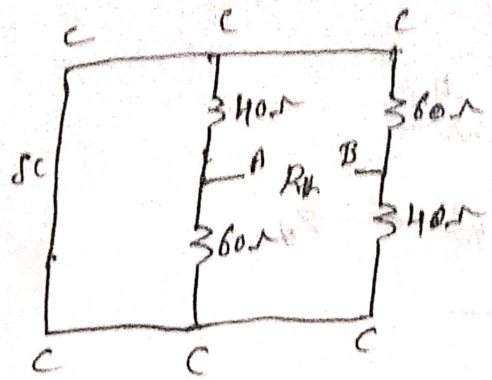
By inspection,

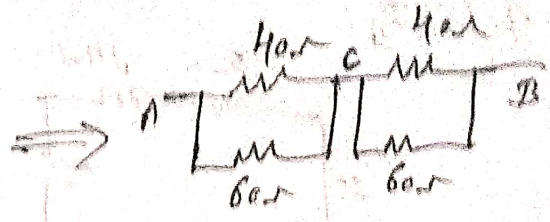
$$(100)I_1 + (-100)I_2 = 60$$
$$(-100)I_1 + (200)I_2 = 0$$
$$I_1 = 1.2A$$
$$I_2 = 0.6A$$

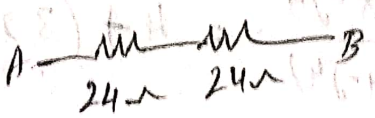


$$40(I_2 - I_1) + 60I_2 - V_{th} = 0$$
$$V_{th} = 40(I_2 - I_1) + 60I_2$$
$$V_{th} = 40(0.6 - 1.2) + 60 \times 0.6$$
$$V_{th} = 12V$$

To find R_{th}:



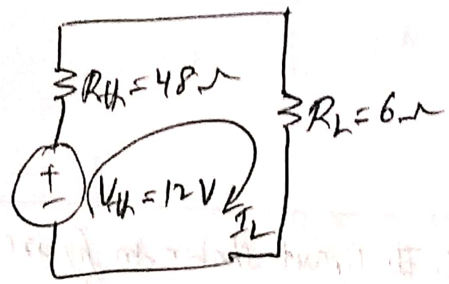




$$\frac{40\times60}{(40+60)} = 24\Omega$$

$$R_{th} = R_{AB} = 48\Omega$$

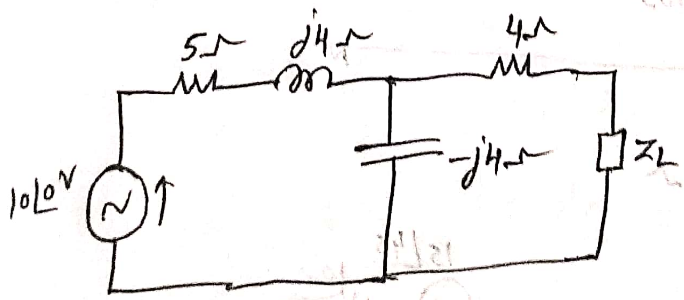
Thevenin equivalent circuit:



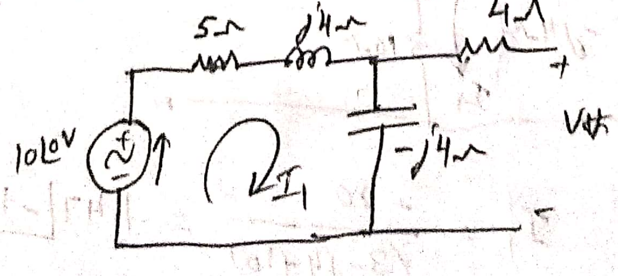
$$I_L = \frac{V_{th}}{R_L + R_{th}} = \frac{12}{(6+48)}$$

$$I_L = 0.222 A$$

Q. Obtain the Thevenin equivalent circuit as seen by the load impedance for the network shown in figure.



Sol: To find V_{th}: Remove Z_L



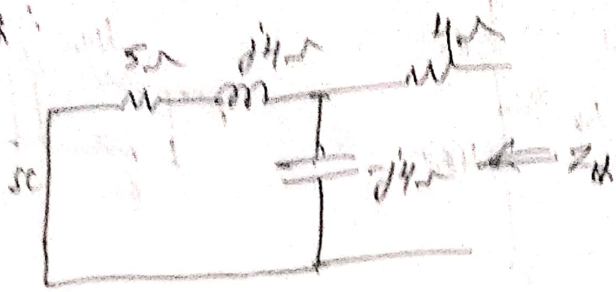
$$I_1 = \frac{10\angle0}{(5+j4-j4)}$$

$$I_1 = 2\angle0\ A$$

$$V_{th} = -j4\times I_1 = -j4\times2\angle0^\circ$$

$$V_{th} = 8\angle-90\ V$$

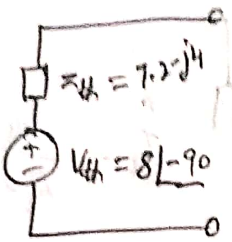
To find Z_{th}:



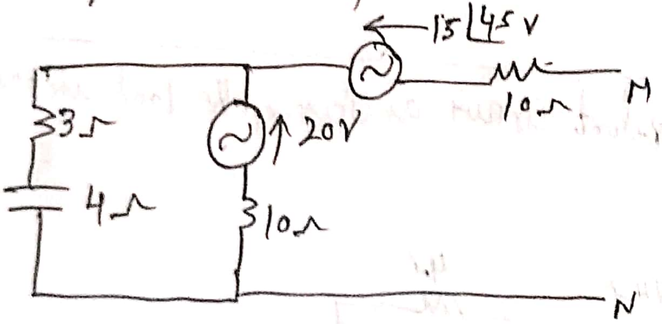
$$Z_{th} = 4 + \frac{(5+j4) \times -j4}{(5+j4-j4)} = 4 + (3.2 - j4)$$

$$Z_{th} = (7.2 - j4)\,\Omega$$

Thevenin eq. ckt

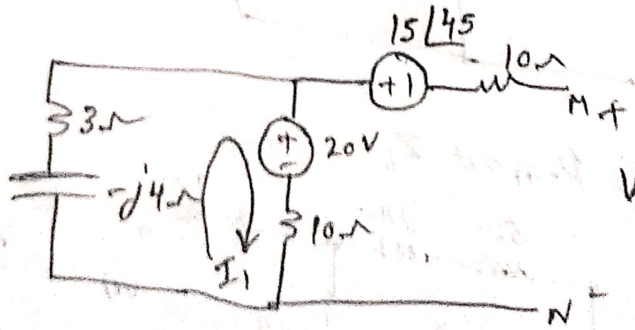


Q. Draw the Thevenin's equivalent circuit for the circuit shown in figure.

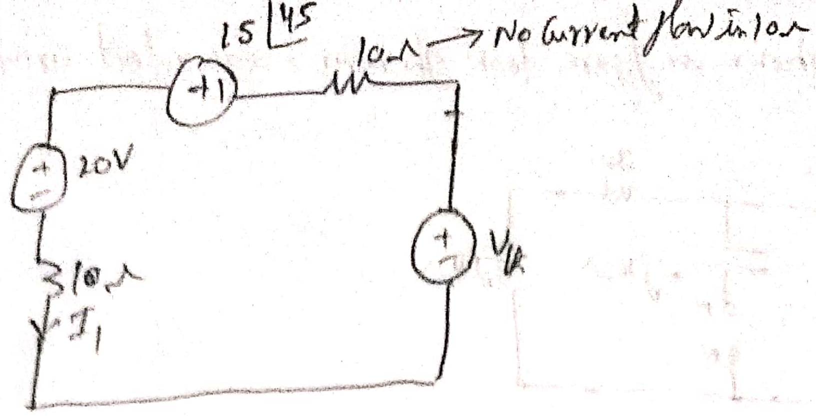


Sol: Attach $-j$ for $X_c \Rightarrow -j4\,\Omega$

To find V_{th}:



$$I_1 = \frac{-20}{(3 - j4 + 10)} = 1.47\angle -162.89 \text{ A}$$



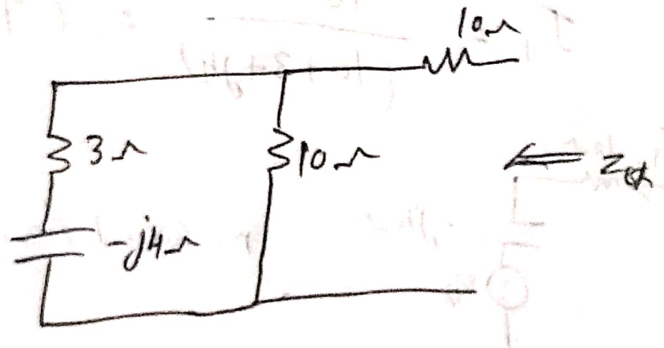
KVL: $-10I_1 - 20 + 15\angle 45 + V_{th} = 0$

$$V_{th} = 20 + 10I_1 - 15\angle 45$$

$$V_{th} = 20 + 10 \times 1.47\angle -162.89 - 15\angle 45$$

$$V_{th} = 15.64\angle -107.3 \text{ V}$$

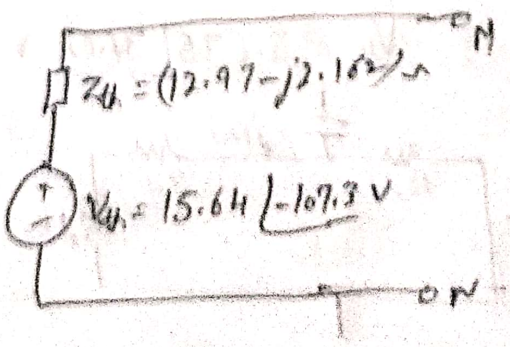
To find Z_{th}:



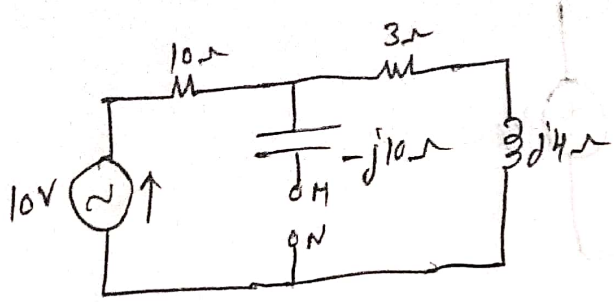
$$Z_{th} = 10 + \frac{(3-j4)\times 10}{(3-j4+10)} = 10 + (2.97 - j2.162)$$

$$Z_{th} = (12.97 - j2.162)\,\Omega$$

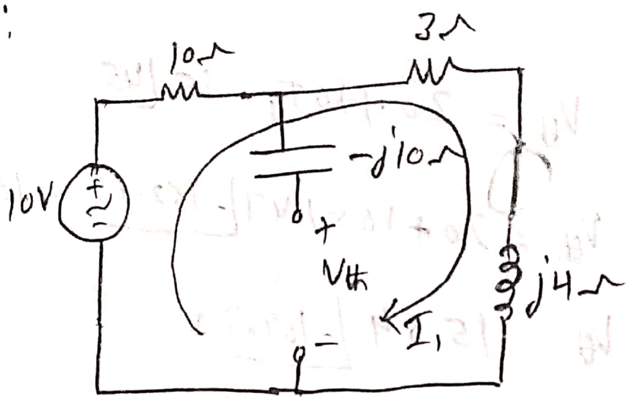
Thevenin Eq. ckt:



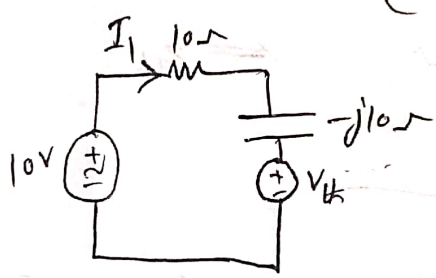
Q. For the network shown in figure draw Thevenin's equivalent circuit.



Sol: <u>To find V_{th}:</u>



$$I_1 = \frac{10}{(10+3+j4)} = 0.7352\angle -17.10 \; A$$



No current flows through -j10Ω

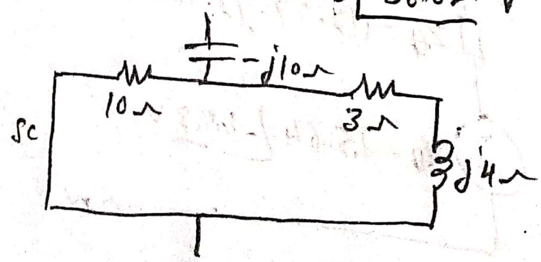
KVL: $-10 + 10 I_1 + V_{th} = 0$

$$V_{th} = 10 - 10 I_1$$

$$V_{th} = 10 - 10 \times 0.7352\angle -17.10$$

$$V_{th} = 3.675\angle 36.02 \; V$$

<u>To find Z_{th}:</u>

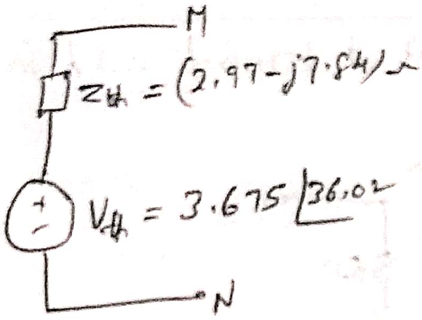


10 & (3+j4) in parallel

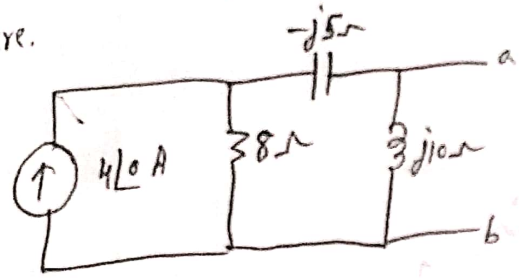
$$Z_{th} = -j10 + \frac{10\times(3+j4)}{(10+3+j4)} = -j10 + (2.97+j2.162)$$

$$Z_{th} = (2.97 - j7.838)\,\Omega$$

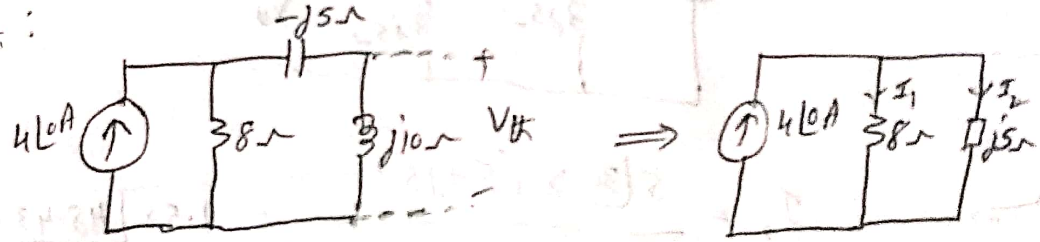
Thevenin eq. ckt :



Q. Find the Thevenin equivalent circuit at the terminals a-b for the circuit shown in figure.



Sol: . To find V_{th} :



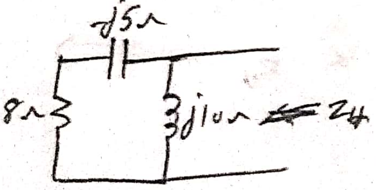
$-j5$ & $j10$ are in series

By current division, $I_2 = \dfrac{4\angle 0 \times 8}{(8+j5)} = 3.39\angle -32$ A

$$V_{th} = j10 \times I_1 = j10 \times 3.39\angle -32$$

$$V_{th} = 33.92\angle 58 \text{ V}$$

To find Z_{th} :



$$Z_{th} = \frac{(8-j5)\times j10}{(8-j5+j10)} = (8.98+j4.38)\,\Omega$$

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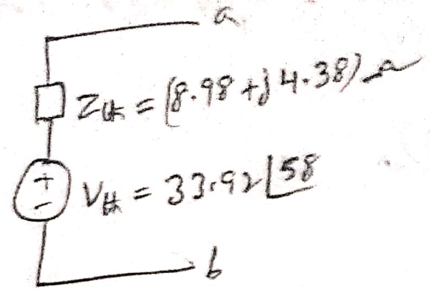
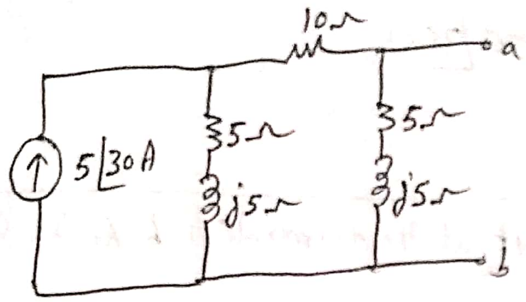
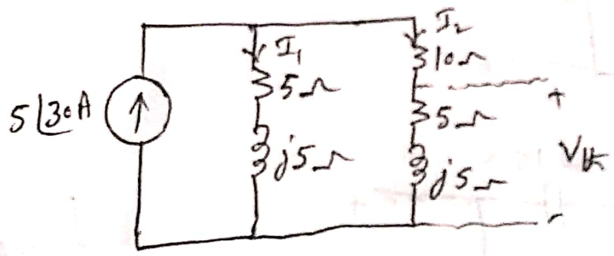
Thevenin Eq. ckt:



Q. Obtain the Thevenin equivalent circuit at terminals a & b for the network shown in figure.



Sol: To find V_{th}:

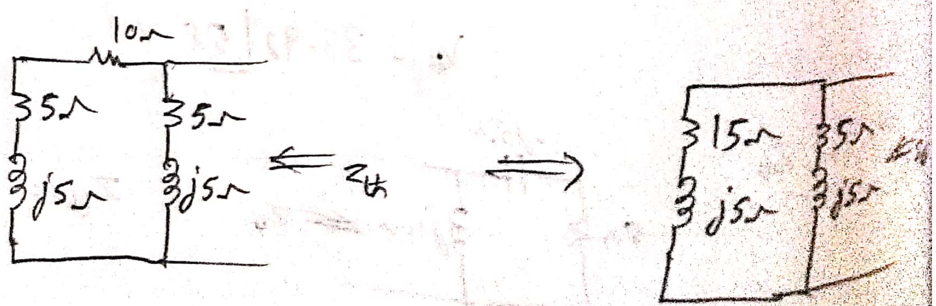


$$I_2 = \frac{5\angle 30 \times (5+j5)}{(5+j5+10+5+j5)} = 1.58\angle 48.43\ A$$

$$V_{th} = (5+j5) \times I_2 = (5+j5) \times 1.58\angle 48.43$$

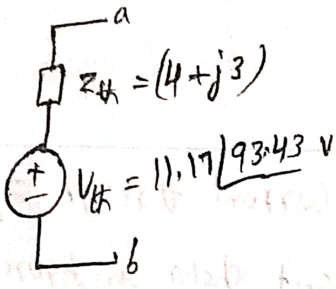
$$V_{th} = 11.17\angle 93.43\ V$$

To find Z_{th}:

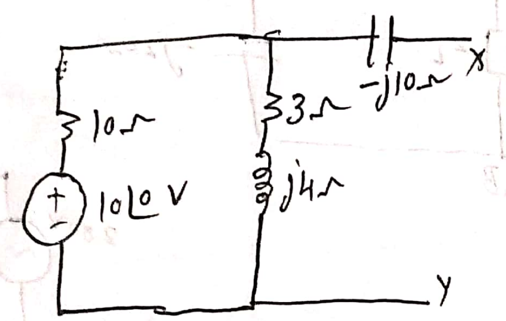


$$Z_{th} = \frac{(15+j5) \times (5+j5)}{(15+j5+5+j5)} = (4+j3)\Omega$$

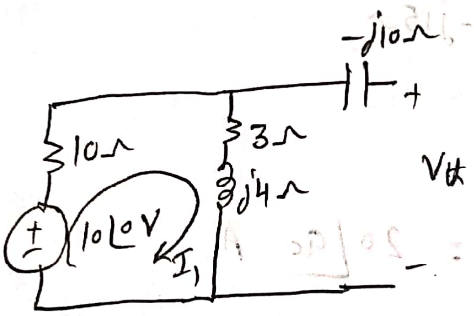
Thevenin equivalent ckt:



Q. obtain the Thevenin's equivalent Circuit for the network shown in figure.



Sol: To find V_{th}:



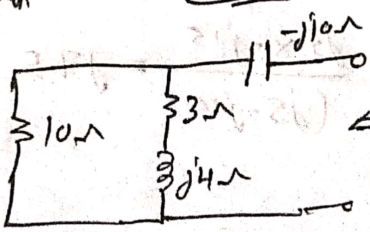
$$I_1 = \frac{10\angle 0}{(10+3+j4)} = 0.7352\angle -17.10 \text{ A}$$

$V_{th} = (3+j4)\, I_1$ (No current flows in −j10Ω)

$V_{th} = (3+j4) \times 0.7352\angle -17.10$

$V_{th} = 3.676\angle 36.03$ V

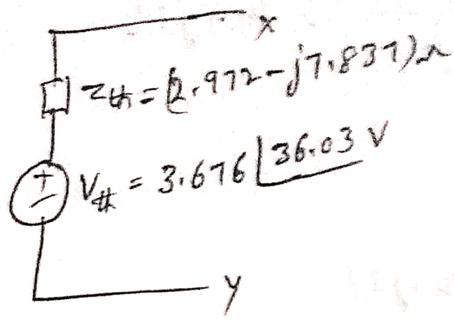
To find Z_{th}:



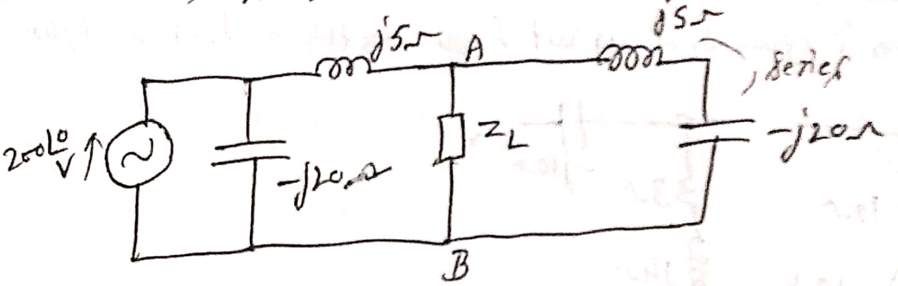
$$Z_{th} = \frac{10 \times (3+j4)}{(10+3+j4)} - j10$$

$Z_{th} = (2.972 - j7.837)\Omega$

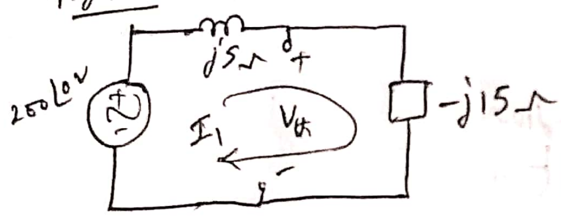
Thevenin eq. ckt :

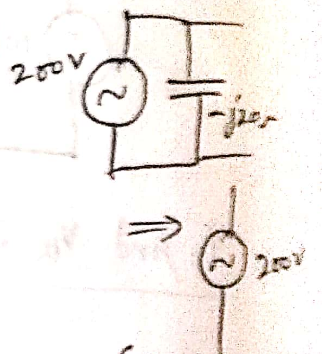


Q. Using Thevenin's theorem find the current through $Z_L = (10 - j7.5)\Omega$ if connected across AB in the circuit shown in figure.



Sol: To find V_{th} :



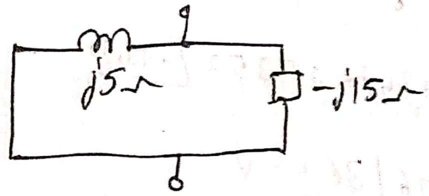


(Impedance across voltage source is neglected)

$$I_1 = \frac{200\angle 0}{(j5 - j15)} = 20\angle 90 \text{ A}$$

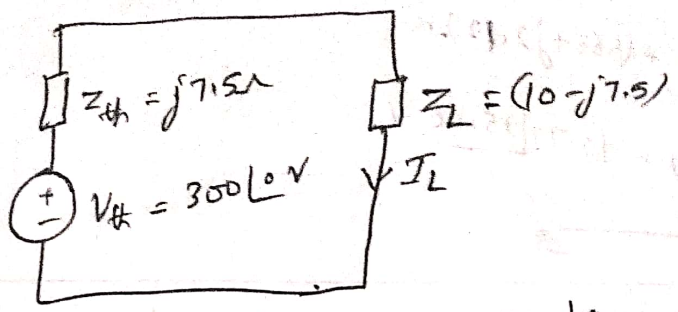
$$V_{th} = -j15 \times I_1 = -j15 \times 20\angle 90 = 300\angle 0 \text{ V}$$

To find Z_{th} :



$$Z_{th} = \frac{j5 \times -j15}{(j5 - j15)} = j7.5$$

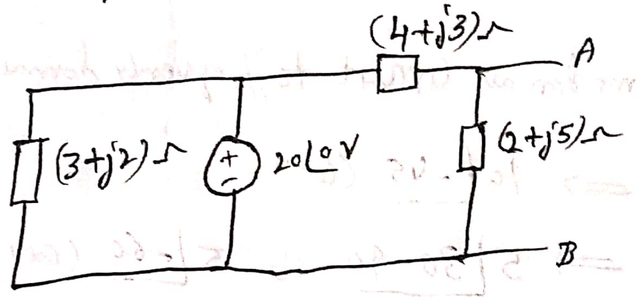
Thevenin Eq. Ckt :



$$I_L = \frac{V_{th}}{(Z_L + Z_{th})} = \frac{300\angle 0}{(10 - j7.5 + j7.5)}$$

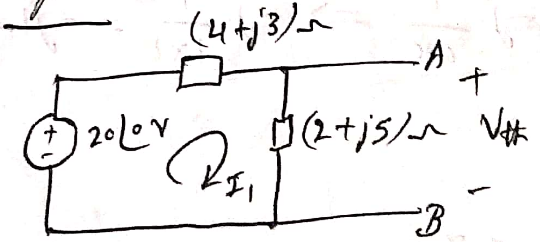
$$I_L = 30\angle 0 \text{ A}$$

Q. obtain the Thevenin equivalent circuit across A-B for the circuit shown in figure.



Sol: $(3+j2)\Omega$ neglected.

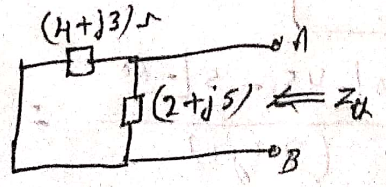
To find V_{th} :



$$I_1 = \frac{20\angle 0}{(4+j3+2+j5)} = 2\angle -53.13 \text{ A}$$

$$V_{th} = (2+j5)I_1 = (2+j5) \times 2\angle -53.13 = 10.77\angle 15.06 \text{ V}$$

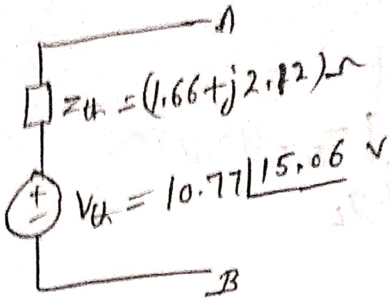
To find Z_{th} :



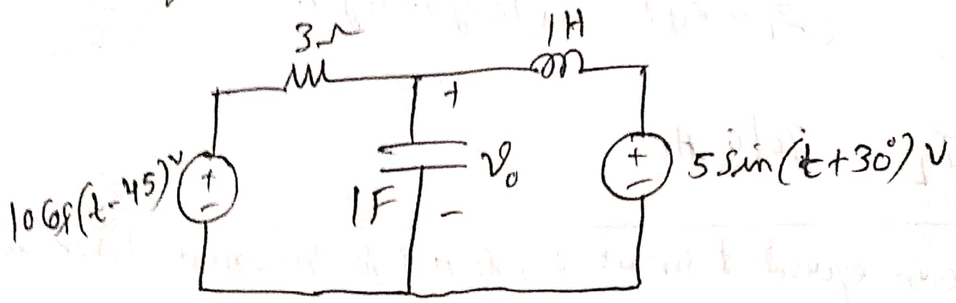
$$Z_{th} = \frac{(4+j3) \times (2+j5)}{(4+j3+2+j5)}$$

$$Z_{th} = (1.66 + j2.12)\Omega$$

Thevenin equivalent Circuit :



Q. Find V_0 using Thevenin's theorem for the Circuit shown in figure.



Sol : Converting the time domain Circuit to frequency domain, $\omega = 1$ (Coefficient of t) in the source

$10\cos(t-45) \Rightarrow 10\angle -45$ (Cos)

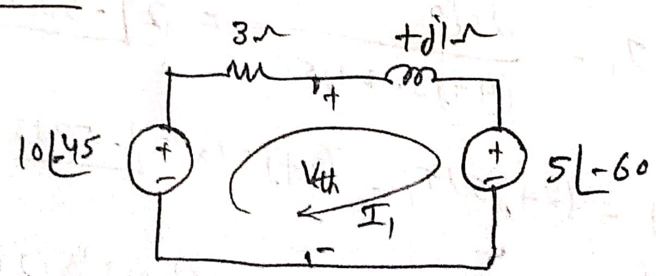
$5\sin(t+30) \Rightarrow 5\angle 30-90 = 5\angle -60$ (Cos)
(Sin to Cos)

$L = 1H \Rightarrow +j\omega L \Rightarrow +j\times 1\times 1 = +j1\Omega$

$C = 1F \Rightarrow -j\cdot\frac{1}{\omega C} \Rightarrow -j\cdot\frac{1}{1\times 1} = -j1\Omega$

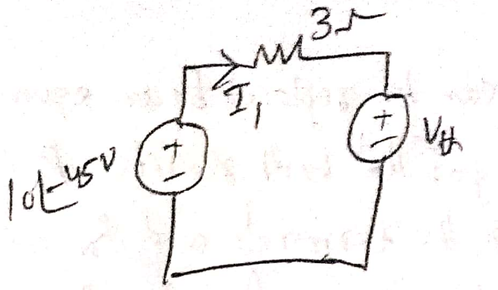
$Z_L = -j1\Omega$

To find V_{th} : Remove Z_L



$$I_1 = \frac{10\angle -45 - 5\angle -60}{(3+j1)} = \frac{(4.57-j2.74)}{(3+j1)}$$

$$I_1 = 1.68\angle{-49.38}\ A$$



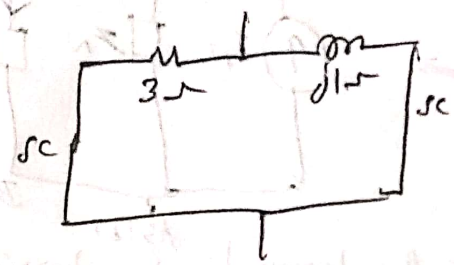
$$-10\angle{-45} + 3I_1 + V_{th} = 0$$

$$V_{th} = 10\angle{-45} - 3I_1$$

$$V_{th} = 10\angle{-45} - 3 \times 1.68\angle{-49.38}$$

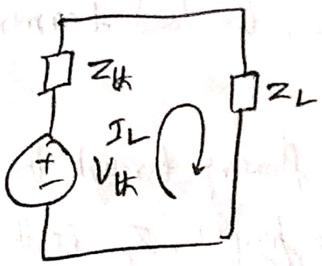
$$V_{th} = 4.98\angle{-40.57}\ V$$

To find Z_{th}:



$$Z_{th} = \frac{3 \times j1}{(3 + j1)} = (0.3 + j0.9)\,\Omega$$

Thevenin Eq. ckt:



$$I_L = \frac{V_{th}}{(Z_L + Z_{th})} = \frac{4.98\angle{-40.57}}{(-j1 + 0.3 + j0.9)} = 15.74\angle{-22.13}\ A$$

To find V_o:

$$V_o = Z_L \times I_L = -j1 \times 15.74\angle{-22.13}$$

$$V_o = 15.74\angle{-112.13}\ V$$

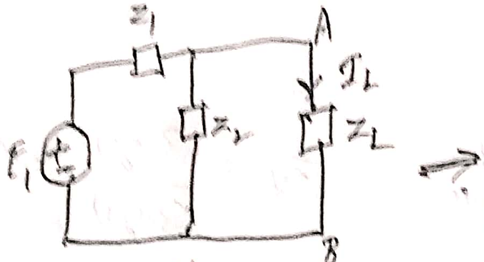
In time domain, $v_o = 15.74 \cos(t - 112.13)\ V$

Norton's theorem

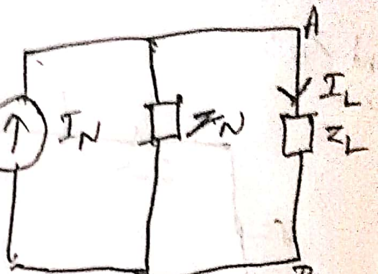
Q. State, explain and prove Norton's theorem.

Statement: A linear two-terminal network can be replaced by an equivalent circuit consisting of a current source I_N in parallel with resistor R_N (or Z_N) where I_N is the short circuit current through the terminals and R_N (or Z_N) is the equivalent resistance (or impedance) at the terminals when the independent sources are turned off or R_N (Z_N) is the ratio of open circuit voltage to short-circuit current at the terminal pair.

Explanation:



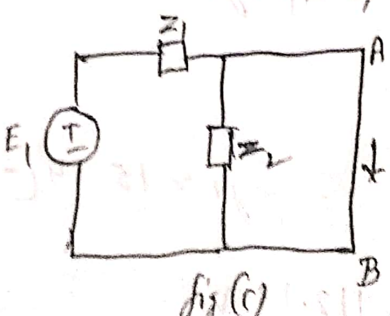
fig(a)



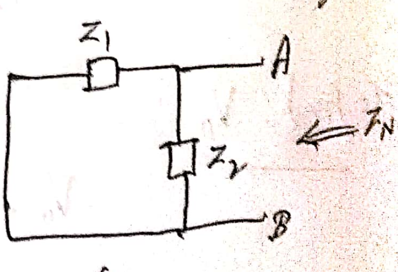
fig(b)

The concept of Norton's equivalent across the terminals of interest is shown in fig(a). The terminals A–B are the terminals where load impedance Z_L is connected. The Norton's equivalent across the load terminals A–B is shown in fig(b).

The Norton current, I_N is the current flowing through the short which is the result of shorting Z_L. This is shown in fig.(c). Z_N is the equivalent impedance as viewed through terminals A–B, with Z_L removed and voltage sources replaced by short circuit and current sources by open circuit. This is shown in fig(d).



fig(c)



fig(d)

While obtaining I_N, any of the network simplification techniques may be used. By the Norton equivalent circuit in fig(b)

$$I_L = \frac{I_N Z_N}{Z_L + Z_N}$$

Proof: Consider the network shown in fig (e). Let us obtain the current through the impedance Z_3 by mesh analysis first.

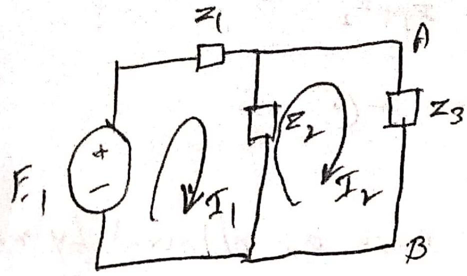


fig (e)

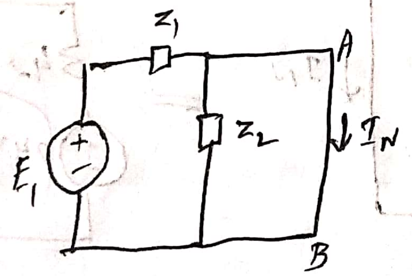
$$(Z_1+Z_2)I_1 + (-Z_2)I_2 = E_1$$
$$(-Z_2)I_1 + (Z_2+Z_3)I_2 = 0$$

$$I_L = I_2 = \frac{\begin{vmatrix} (Z_1+Z_2) & E_1 \\ -Z_2 & 0 \end{vmatrix}}{\begin{vmatrix} (Z_1+Z_2) & -Z_2 \\ -Z_2 & (Z_2+Z_3) \end{vmatrix}} = \frac{E_1 Z_2}{Z_1Z_2 + Z_1Z_3 + \cancel{Z_2^2} + Z_2Z_3 - \cancel{Z_2^2}}$$

$$I_L = I_2 = \frac{E_1 Z_2}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} \quad —① $$

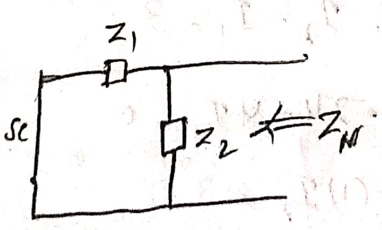
Now I_L is obtained by Norton's theorem

To find I_N:



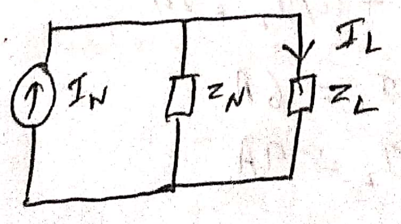
$$I_N = \frac{E_1}{Z_1}$$ (Z_2 is removed since it is across short circuit)

To find Z_N:



$$Z_N = \frac{Z_1 Z_2}{(Z_1+Z_2)}$$

To find I_L:



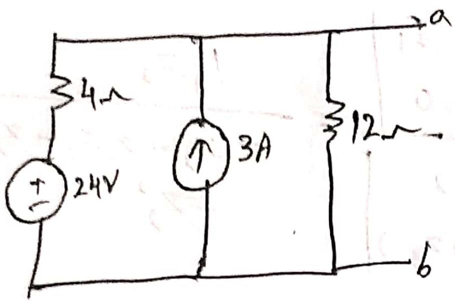
$Z_L = Z_3$

$$I_L = \frac{I_N Z_N}{Z_L + Z_N} = \frac{\frac{E_1}{Z_1} \times \frac{Z_1 Z_2}{Z_1 + Z_2}}{Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2}} = \frac{E_1 Z_2}{Z_1 Z_3 + Z_2 Z_3 + Z_1 Z_2}$$

$$I_L = \frac{E_1 Z_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} \quad \text{--- (2)}$$

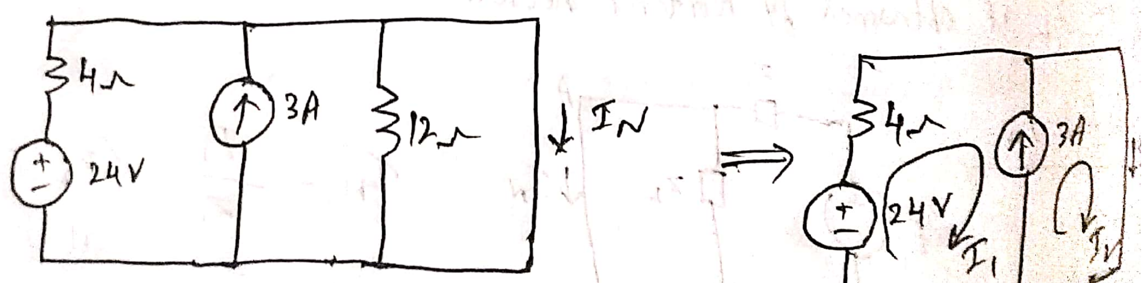
Comparing equation (1) and (2), I_L is same as obtained by mesh analysis. Thus, the Norton's theorem is verified (proved).

Q. Find the Norton's equivalent circuit across terminals a, b of the network shown in figure.



Sol: To find I_N (or I_{sc} → short circuit current), R_L is not given

Terminal a-b is short circuited



KCL to supermesh 1 & 2 : $I_2 - I_1 = 3$

KVL to supermesh 1 & 2 : $-24 + 4I_1 = 0$

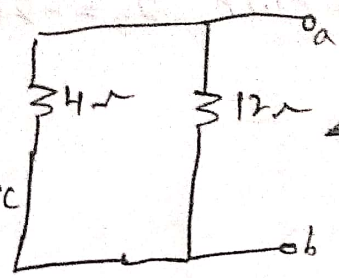
$$(-1)I_1 + (1)I_2 = 3$$

$$(4)I_1 + (0)I_2 = 24$$

$$I_1 = 6\,A$$

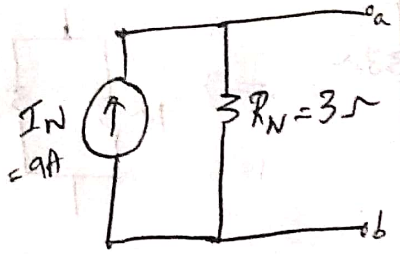
$$I_N = I_2 = 9\,A$$

To find R_N:

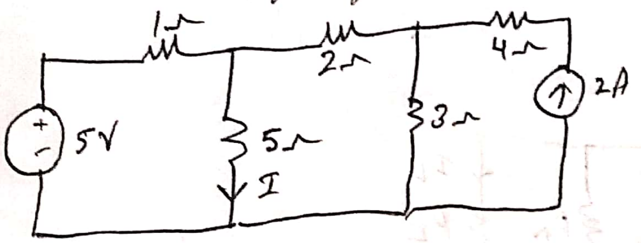


$R_N = \frac{4\times12}{(4+12)} = 3\Omega$

Norton equivalent circuit:

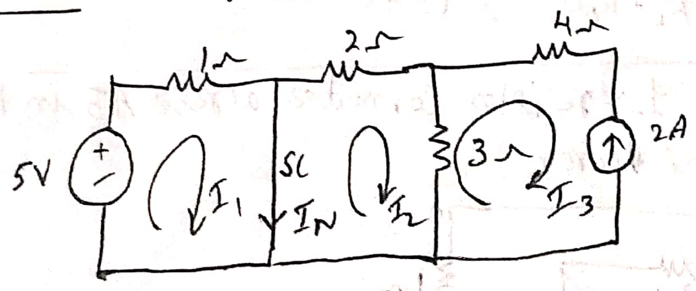


Q. For the network shown in figure, find current I using Norton's theorem.



Sol: $R_L = 5\Omega$

To find I_N: Replace R_L by short circuit



KCL for non-essential mesh 3: $I_3 = -2$

KVL for essential mesh 1: $-5 + 1I_1 = 0$

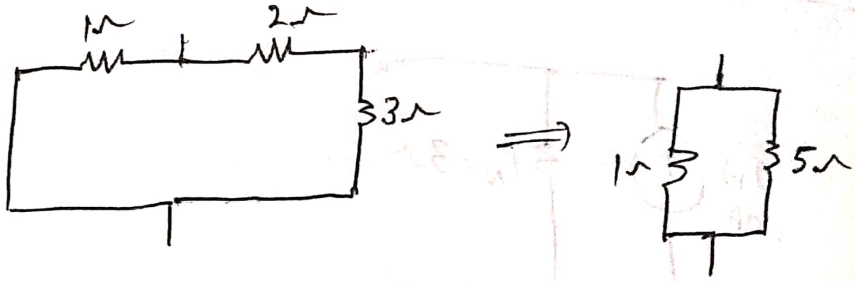
KVL for essential mesh 2: $2I_2 + 3(I_2 - I_3) = 0$

$$(0)I_1 + (0)I_2 + (1)I_3 = -2$$
$$(1)I_1 + (0)I_2 + (0)I_3 = 5$$
$$(0)I_1 + (5)I_2 + (-3)I_3 = 0$$

$I_1 = 5A$, $I_2 = -1.2A$, $I_3 = -2A$

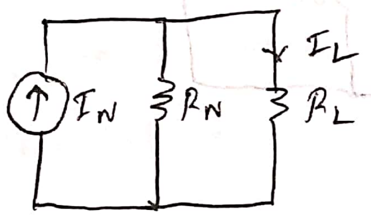
$$I_N = I_1 - I_2$$
$$= 5 - (-1.2)$$
$$I_N = 6.2\,A$$

To find R_N:



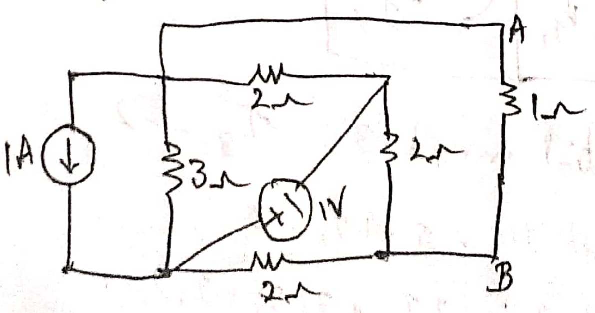
$$R_N = \frac{1 \times 5}{(1+5)} = 0.833A$$

Norton equivalent circuit:

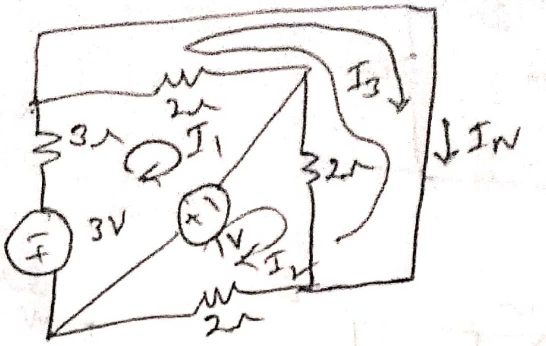


$$I_L = \frac{I_N R_N}{(R_L + R_N)} = \frac{6.2 \times 0.833}{(5+0.833)} = 0.885\,A$$

Q. Determine the current through 1Ω resistor connected across AB in the network shown in figure using Norton's theorem.



Sol: To find I_N: Replace 1Ω by short circuit



$$(5)I_1 + (0)I_2 + (-2)I_3 = -3+1$$

$$(0)I_1 + (4)I_2 + (-2)I_3 = -1$$

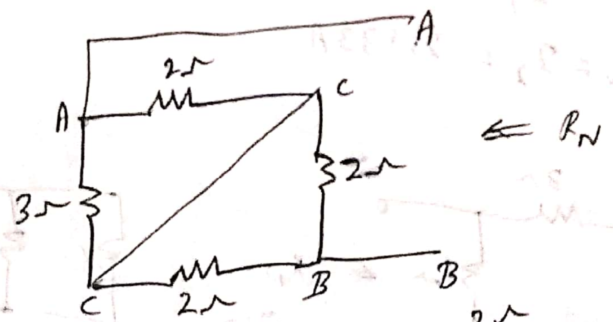
$$(-2)I_1 + (-2)I_2 + (4)I_3 = 0$$

$$I_1 = -0.636A$$

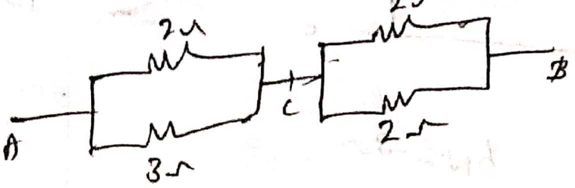
$$I_2 = -0.545A$$

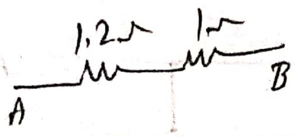
$$I_{sc} = I_N = -0.5909A$$

To find R_{th}:



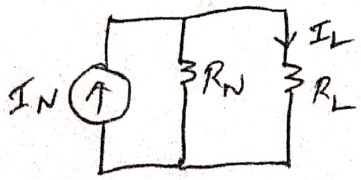
Rewriting the circuit





$$R_N = 2.2\Omega$$

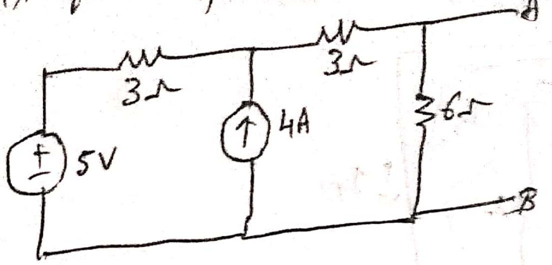
Norton equivalent circuit:



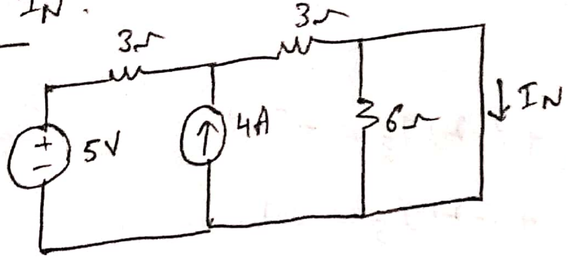
$$I_L = \frac{I_N R_N}{R_L + R_N} = \frac{-0.5909 \times 2.2}{(1+2.2)}$$

$$I_L = -0.406A$$

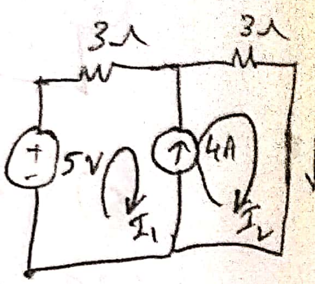
Q. Find the Norton's equivalent for the circuit shown in figure.



Sol: To find I_N:



$\Longrightarrow$



KCL equation for supermesh 1 & 2: $I_2 - I_1 = 4$

KVL equation for supermesh 1 & 2: $-5 + 3I_1 + 3I_2 = 0$

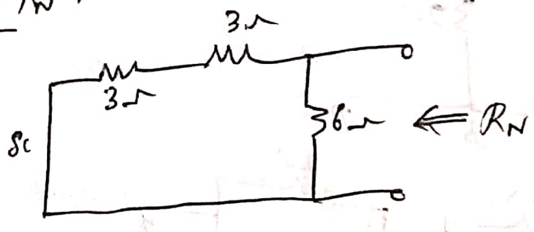
$$(-1)I_1 + (1)I_2 = 4$$

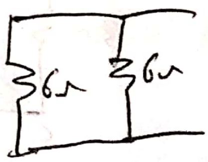
$$(3)I_1 + (3)I_2 = 5$$

$$I_1 = -1.166\ A$$

$$I_N = I_2 = 2.833\ A$$

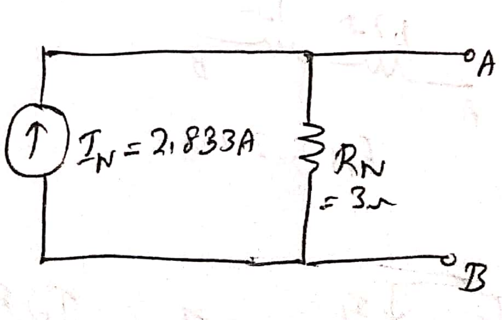
To find R_N:



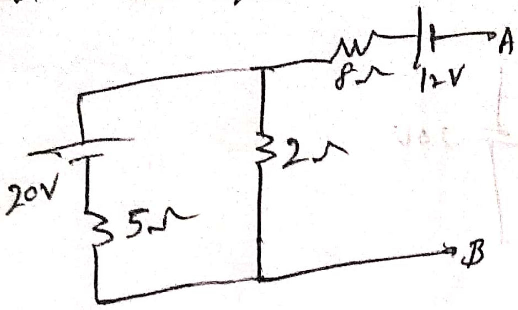


$$R_N = 3\Omega$$

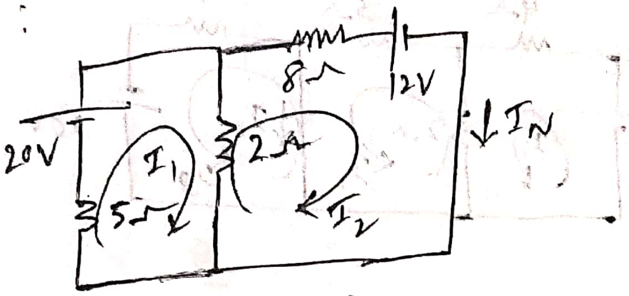
Norton Eqi. ckt:



6. obtain the Norton equivalent circuit for the circuit shown in figure.



Sol: **To find I_N:**



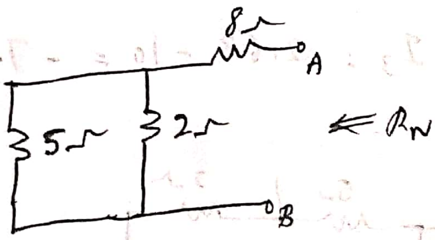
By inspection, $(7)I_1 + (-2)I_2 = 20$

$(-2)I_1 + (10)I_2 = -12$

$I_1 = 2.66\ A$

$I_N = I_2 = -0.666\ A$

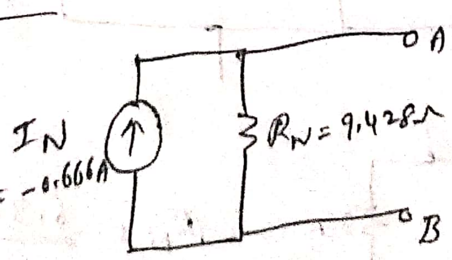
To find R_N:



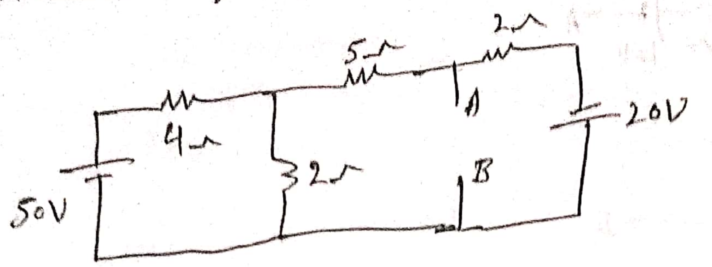
$R_N = \frac{5 \times 2}{(5+2)} + 8$

$R_N = 9.428\ \Omega$

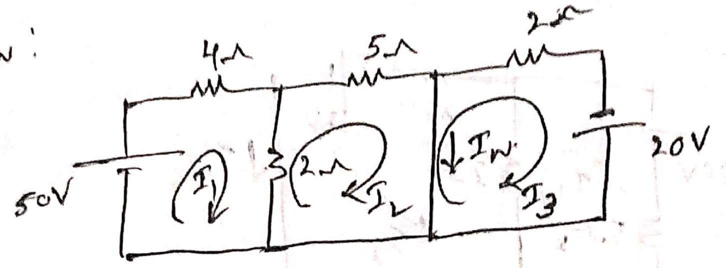
Norton equivalent circuit:



Q. obtain the Norton equivalent circuit for the network shown in figure.



Sol: To find I_N:

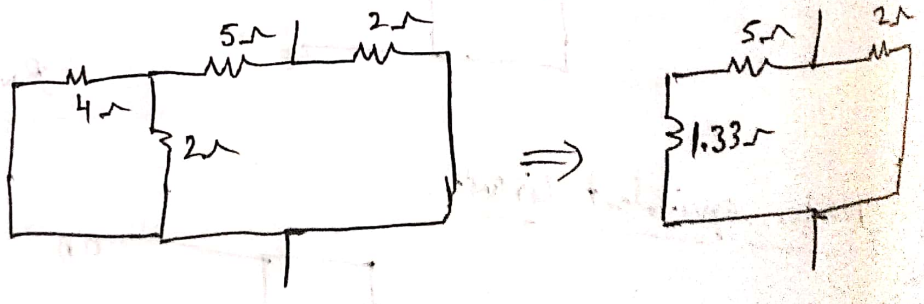


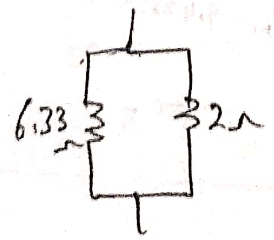
$$(6)I_1 + (-2)I_2 + (-0)I_3 = 50$$
$$(-2)I_1 + (7)I_2 + (0)I_3 = 0$$
$$(-0)I_1 + (-0)I_2 + (2)I_3 = 20$$

$$I_1 = 9.21\ A$$
$$I_2 = 2.63\ A$$
$$I_3 = 10A$$

$$I_N = I_2 - I_3 = 2.63 - 10 = -7.37A$$

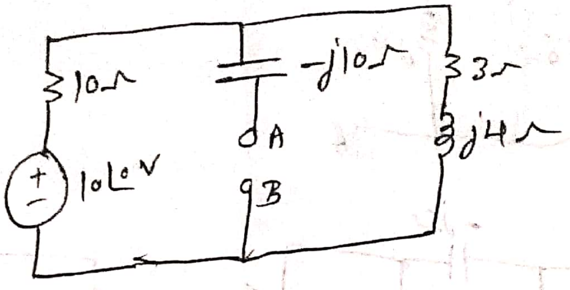
To find R_N:



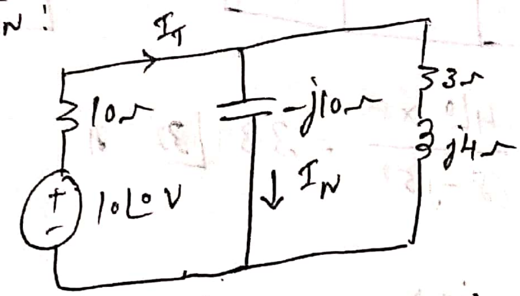


$$R_N = \frac{6.33 \times 2}{(6.33+2)} = 1.519\Omega$$

Q. Find the Norton's equivalent circuit at the terminals A&B for the network shown in figure.



Sol: To find I_N:



Total impedance, $Z_T = 10 + \dfrac{(3+j4)\times -j10}{(3+j4-j10)}$

$$= 10 + 6.66 + j3.33$$

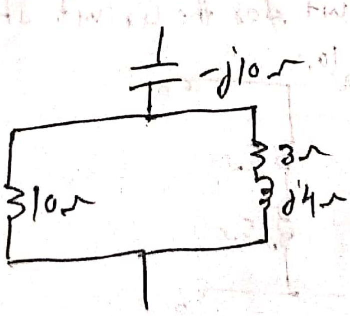
$$= 16.66 + j3.33$$

$$I_T = \frac{V_T}{Z_T} = \frac{10\angle 0}{(16.66+j3.33)} = 0.588\angle -11.30 \text{ A}$$

By current division, $I_N = \dfrac{0.588\angle -11.30 \times (3+j4)}{(3+j4-j10)}$

$$I_N = 0.438\angle 105.2 \text{ A}$$

To find Z_N:

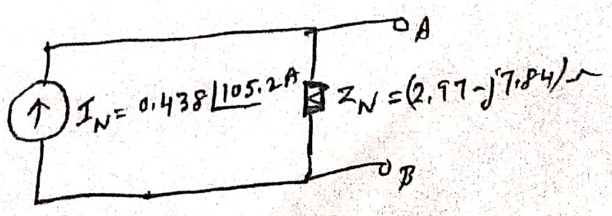


$$Z_N = \frac{10\times(3+j4)}{(10+3+j4)} - j10$$

$$Z_N = 2.97 + j2.16 - j10$$

$$Z_N = (2.97 - j7.84)\,\Omega$$

Norton equivalent circuit:



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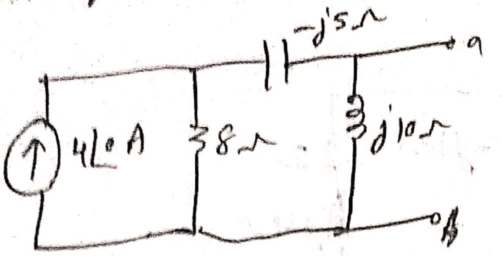
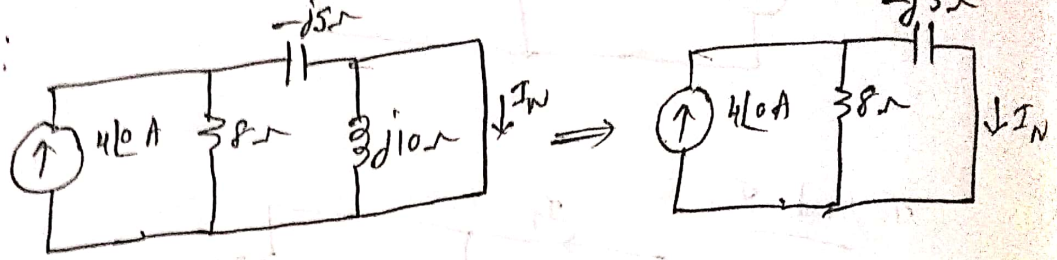
Q. Find the Norton equivalent circuit for the network shown in figure.

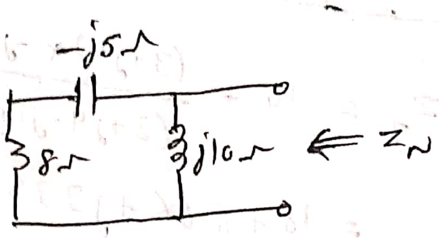


Sol: To find I_N:



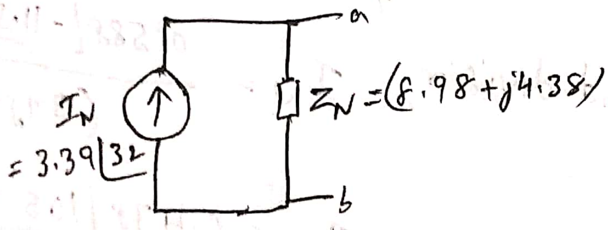
By current division,
$$I_N = \frac{4\angle 0 \times 8}{(8-j5)} = 3.39\angle 32 \text{ A}$$

To find Z_N:

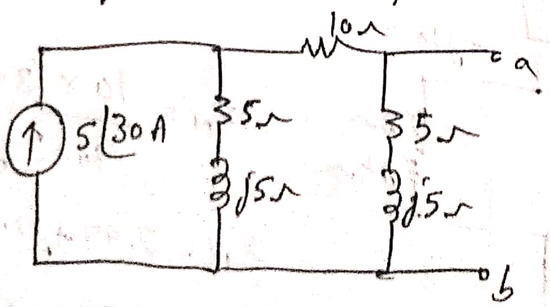


$$Z_N = \frac{(8-j5)\times j10}{(8-j5+j10)} = (8.98+j4.38)\Omega$$

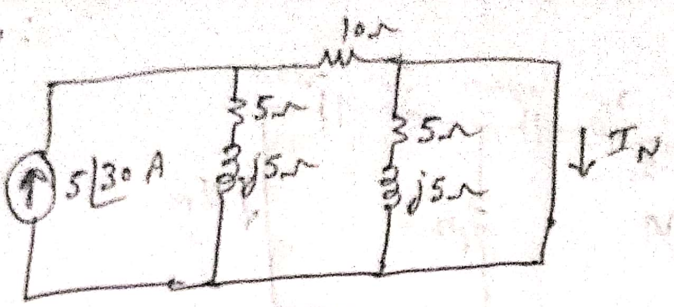
Norton Eq. ckt:

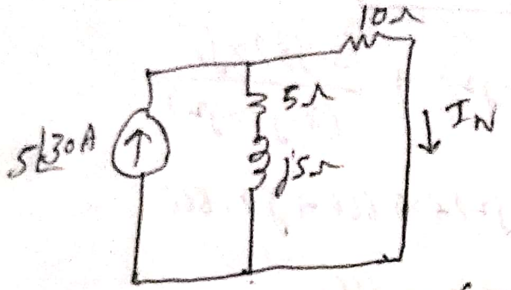


Q. Obtain the Norton equivalent circuit for the circuit shown in figure.



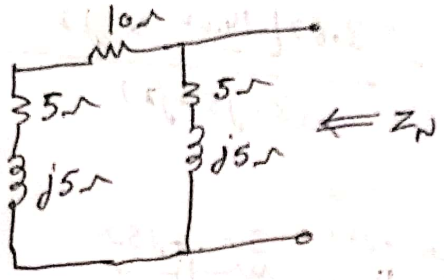
Sol: To find I_N:





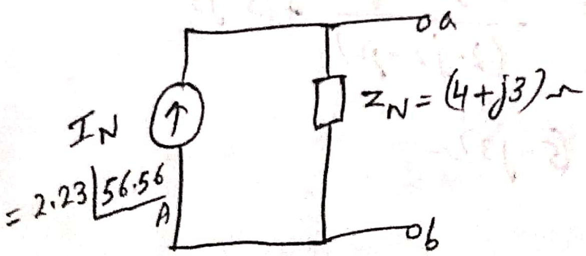
$$I_N = \frac{5\angle 30 \times (5+j5)}{(5+j5+10)} = 2.23\angle 56.56\ A$$

To find Z_N:

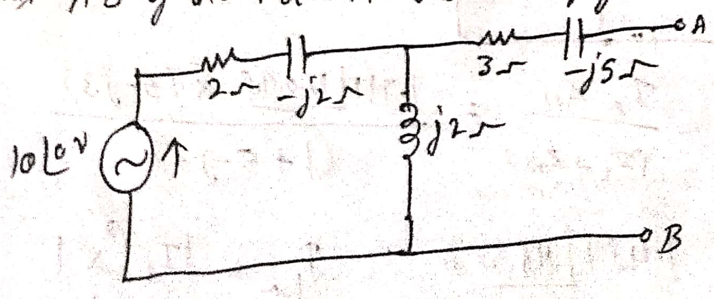


$$Z_N = \frac{(15+j5)\times(5+j5)}{(15+j5+5+j5)} = (4+j3)\,\Omega$$

Norton equivalent circuit:



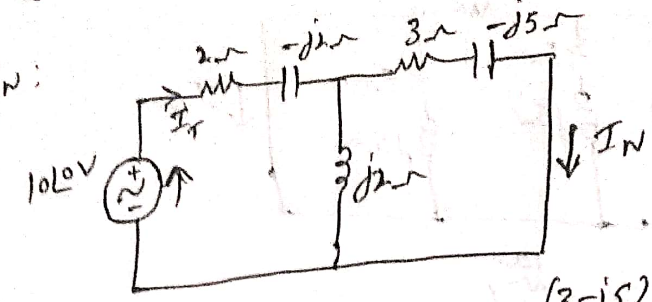
Q. Use Norton's theorem to find the power in a 1 ohm resistor connected to terminals AB of the network shown in figure.



Sol:

Given: $Z_L = 1\Omega$

To find I_N:



Total impedance, $Z_T = (2-j2) + \dfrac{(3-j5)\times j2}{(3-j5+j2)}$

$Z_T = (2-j2) + 0.666 + j2.666$

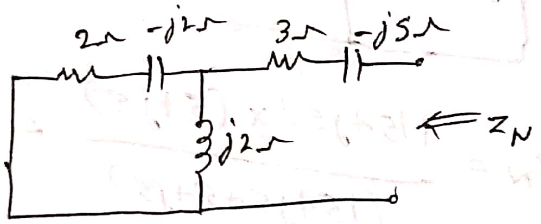
$Z_T = 2.666 + j0.666$

$I_T = \dfrac{V_T}{Z_T} = \dfrac{10\angle 0}{(2.666+j0.666)} = 3.63\angle -14.02\ A$

$I_N = \dfrac{3.63\angle -14.02 \times j2}{(3-j5+j2)} = 1.711\angle 120.98\ A$

or find I_N using mesh analysis writing equations by inspection method & finding $I_2 = I_N$ by Cramer's rule.

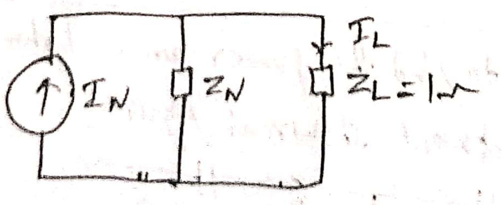
To find Z_N:



$Z_N = \dfrac{(2-j2)\times j2}{(2-j2+j2)} + (3-j5)$

$Z_N = (5-j3)\Omega$

Norton equivalent circuit:



$I_L = \dfrac{I_N Z_N}{(Z_L + Z_N)} = \dfrac{1.711\angle 120.98 \times (5-j3)}{(1+5-j3)}$

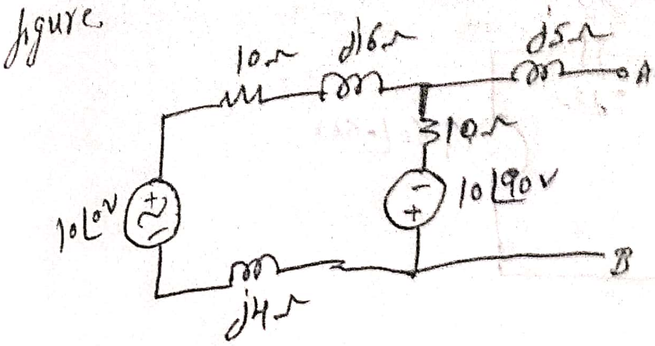
$I_L = 1.487\angle 116.58\ A$

$P_{1\Omega} = |I_L|^2 \times 1$

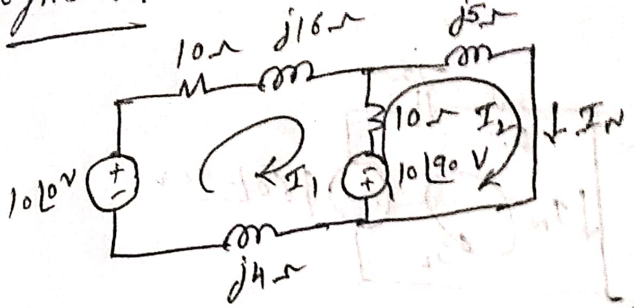
$= 1.487^2 \times 1$

$= 2.21\ W$

a. Find the Norton equivalent Circuit at terminals AB of the network shown in figure.



Sol: To find I_N:



$$(20+j20)I_1 + (-10)I_2 = \underbrace{10\angle 0 + 10\angle 90}_{14.14\angle 45}$$

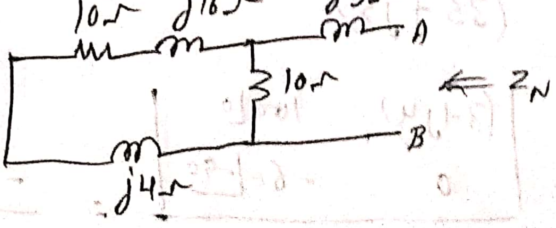
$$(-10)I_1 + (10+j5)I_2 = -10\angle 90$$

$$I_N = I_2 = \frac{\begin{vmatrix} (20+j20) & 14.14\angle 45 \\ -10 & -10\angle 90 \end{vmatrix}}{\begin{vmatrix} (20+j20) & -10 \\ -10 & (10+j5) \end{vmatrix}} = \frac{(20+j20)\times 10\angle -90 + 10\times 14.14\angle 45}{(20+j20)\times(10+j5) - 100}$$

sign removed by adding −180 to angle

$$I_N = \frac{(300 - j100)}{j300} = 1.05\angle -108.4 \text{ A}$$

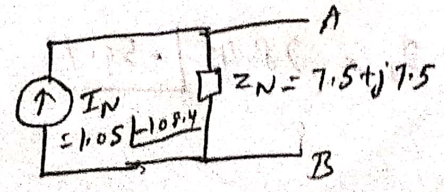
To find Z_N:



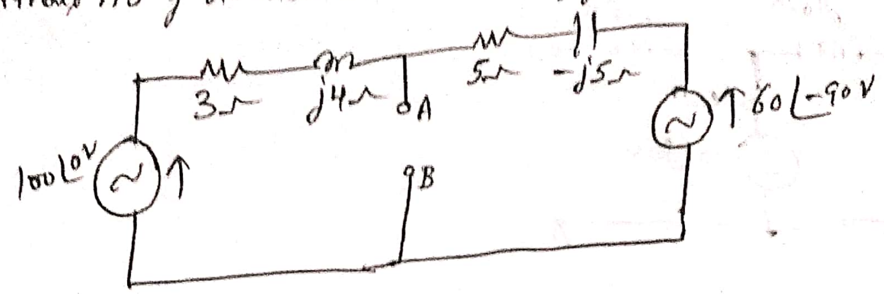
$$Z_N = \frac{(10+j20)\times 10}{(10+j20+10)} + j5$$

$$Z_N = 7.5 + j7.5$$

Norton Equivalent Circuit:

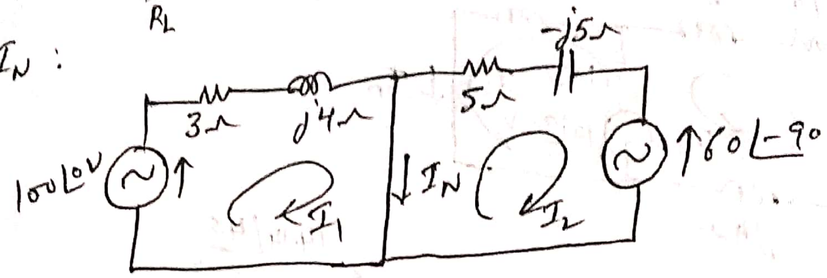


Q. Use Norton's theorem to find the power in an impedance of $(2+j4)\Omega$ connected to terminals AB of the network shown in figure.



Sol: Given: $Z_L = (2+j4)\Omega$
$\downarrow$
R_L

To find I_N :



$$(3+j4)I_1 + (0)I_2 = 100\angle 0$$

$$(0)I_1 + (5-j5)I_2 = -60\angle -90$$

$$I_1 = \frac{\begin{vmatrix} 100\angle 0 & 0 \\ -60\angle -90 & (5-j5) \end{vmatrix}}{\begin{vmatrix} (3+j4) & 0 \\ 0 & (5-j5) \end{vmatrix}} = \frac{100 \times (5-j5)}{(3+j4)\times(5-j5)}$$

$$I_1 = \frac{(500-j500)}{(35+j5)} = 20\angle -53.13 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} (3+j4) & 100\angle 0 \\ 0 & -60\angle -90 \end{vmatrix}}{(35+j5)} = \frac{-(3+j4)\times 60\angle -90}{(35+j5)}$$

$$I_2 = \frac{(-240+j180)}{(35+j5)} = 8.48\angle 135$$

$$I_N = I_1 - I_2 = 28.42\angle -50.71$$

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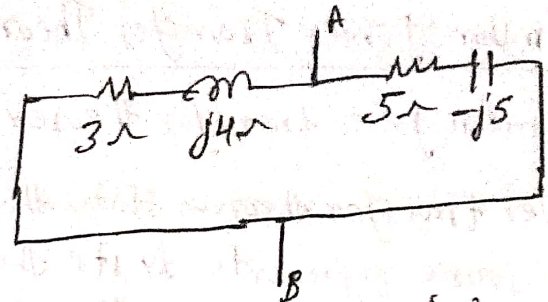
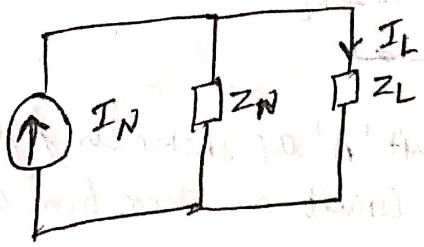
To find Z_N:



$$Z_N = \frac{(3+j4)\times(5-j5)}{(3+j4+5-j5)} = 4.23 + j1.153$$

Norton Eq. ckt



$$I_L = \frac{I_N Z_N}{(Z_L + Z_N)} = \frac{28.42\angle -50.71 \times (4.23 + j1.153)}{(2 + j4 + 4.23 + j1.153)}$$

$$I_L = \frac{(101.48 - j72.29)}{(6.23 + j5.153)} = 15.41\angle -75.06 \text{ A}$$

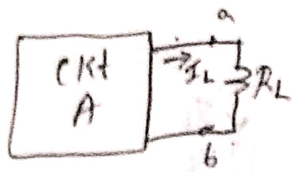
$$P_L = |I_L|^2 R_L = 15.41^2 \times 2 = 475 \text{W}$$

Maximum power Transfer Theorem (DC Circuits)

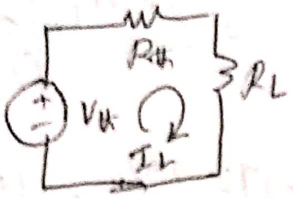
Q. State, explain and prove maximum power transfer theorem for DC circuits

Sol: **Statement**: The maximum power transfer theorem states that the maximum power delivered by a source represented by its thevenin equivalent circuit is attained when the load R_L is equal to the thevenin resistance R_{th}.

Explanation:



Consider a linear circuit 'A' as shown in figure. Circuit 'A' is replaced by its thevenin equivalent circuit as seen from a & b.



$$I_L = \frac{V_{th}}{R_L + R_{th}}$$

The power is delivered to the load is given by

$$P = I_L^2 R_L = \frac{V_{th}^2}{(R_L + R_{th})^2} R_L$$

Proof: Assuming that V_{th} & R_{th} are fixed for a given source, the max. power is a function of R_L. In order to determine the value of R_L that maximizes P, we differentiate p with respect to R_L and equate the derivative to zero.

$$P = \frac{V_{th}^2}{(R_L + R_{th})^2} R_L$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dP}{dR_L} = 0$$

Here $dx = dR_L$
$u = R_L$
$v = (R_L + R_{th})^2$

$$V_{th}^2 \left[\frac{(R_L + R_{th})^2 \times 1 - R_L\{2(R_L + R_{th})\}}{(R_L + R_{th})^4} \right] = 0 \qquad V_{th} \neq 0$$

$$(R_L+R_{th})^2 - 2R_L(R_L+R_{th}) = 0$$

$$R_L^2 + R_{th}^2 + \cancel{2R_LR_{th}} - 2R_L^2 - \cancel{2R_LR_{th}} = 0$$

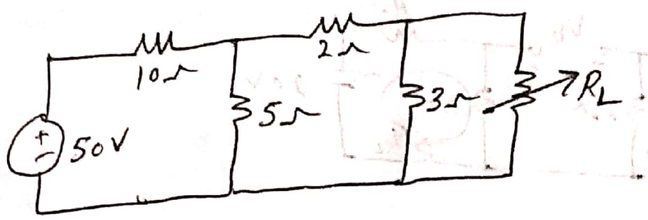
$$-R_L^2 + R_{th}^2 = 0$$

$$R_L^2 = R_{th}^2$$

$$\boxed{R_L = R_{th}}$$

$$P_{max} = I_L^2 R_{th} = \frac{V_{th}^2 \times R_{th}}{(R_{th}+R_{th})^2} = \frac{V_{th}^2}{4R_{th}}$$

Q. Find the maximum power delivered to the load R_L in the circuit shown in figure and hence obtain maximum power.



Sol: To find V_{th}: Remove R_L

$$(15)I_1 + (-5)I_2 = 50$$

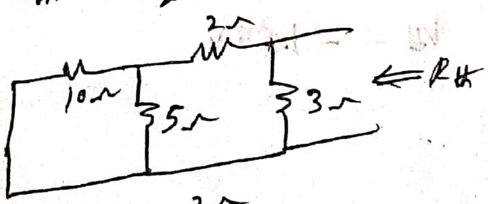
$$(-5)I_1 + (10)I_2 = 0$$

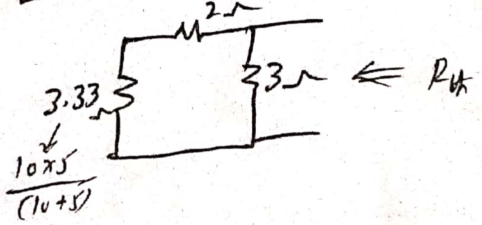
$$I_1 = 4A$$

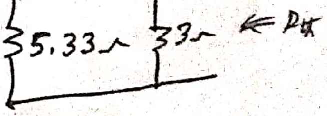
$$I_2 = 2A$$

$$V_{th} = 3I_2 = 6V$$

To find R_{th}:



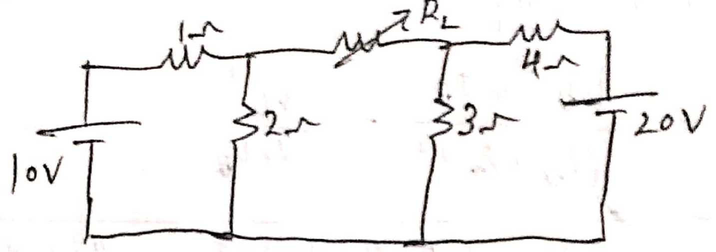




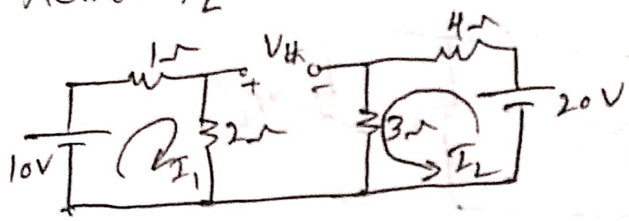
$$R_{th} = \frac{5.33 \times 3}{(5.33+3)} = 1.919\,\Omega \qquad R_L = R_{th} = 1.919\,\Omega$$

$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{6^2}{(4 \times 1.919)} = 4.68\,W$$

Q. Find the value of R_L for which the power transferred across AB of the circuit shown in figure is maximum and the maximum power transferred.

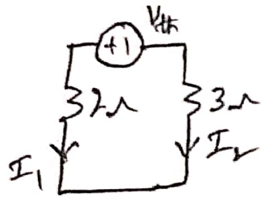


Sol: To find V_{th}: Remove R_L



$$I_1 = \frac{10}{(1+2)} = 3.33\,A$$

$$I_2 = \frac{20}{(4+3)} = 2.85\,A$$



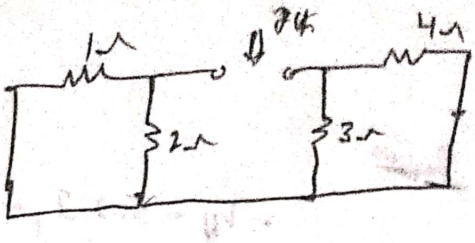
$$-2I_1 + V_{th} + 3I_2 = 0$$

$$V_{th} = 2I_1 - 3I_2$$

$$V_{th} = 2 \times 3.33 - 3 \times 2.85$$

$$V_{th} = -1.89\,V$$

To find R_{th} :



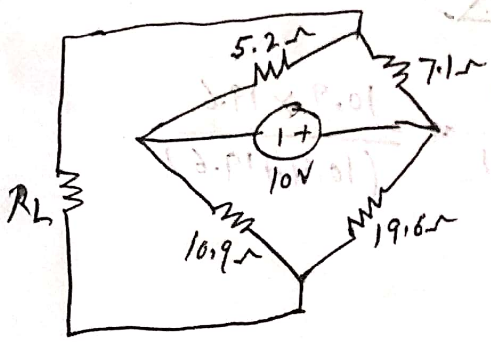
$$R_{th} = \frac{1\times2}{(1+2)} + \frac{3\times4}{(3+4)}$$

$$R_{th} = 2.38\Omega$$

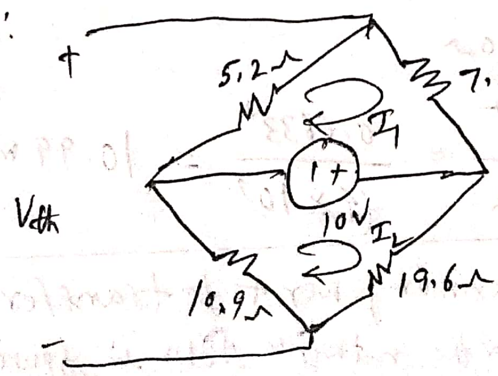
$$R_L = R_{th} = 2.38\Omega$$

$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{(-1.89)^2}{(4\times2.38)} = 0.375W$$

Q. In the circuit shown in figure, find the value of R_L that receives maximum power and find this power.



Sol: To find V_{th} :

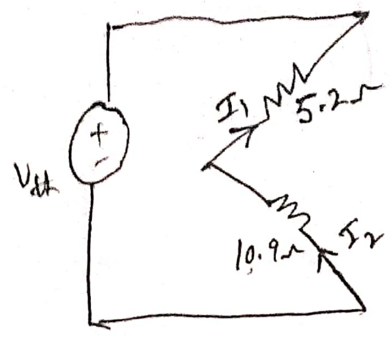


$$(12.3)I_1 + (0)I_2 = -10$$

$$(0)I_1 + (30.5)I_2 = +10$$

$$I_1 = -0.813A$$

$$I_2 = 0.327A$$

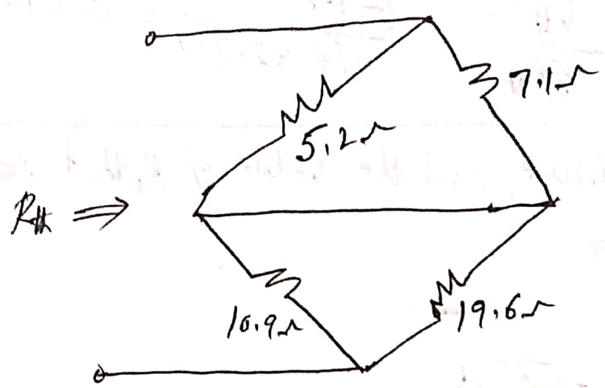


$$-V_{th} - 5.2 I_1 - 10.9 I_2 = 0$$

$$V_{th} = -5.2 \times -0.813 - 10.9 \times 0.327$$

$$V_{th} = 0.6633\,V$$

To find R_{th}:



$$R_{th} = \frac{5.2 \times 7.1}{(5.2+7.1)} + \frac{10.9 \times 19.6}{(10.9+19.6)}$$

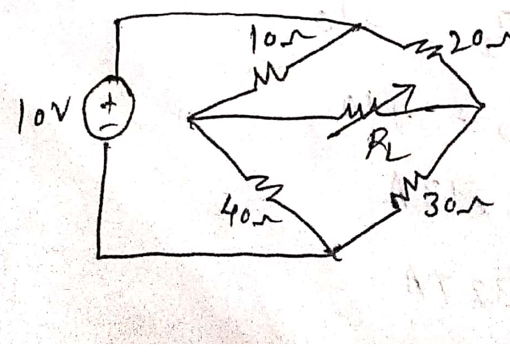
$$R_{th} = 3 + 7$$

$$R_{th} = 10\Omega$$

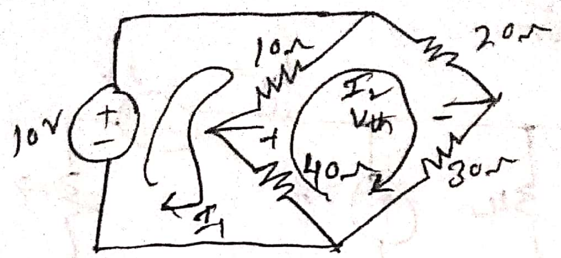
$$R_L = R_{th} = 10\Omega$$

$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{0.6633^2}{(4 \times 10)} = 10.99\,mW$$

Q. Find the value of R_L for maximum power to be transferred to the load and also find maximum power for the network shown in figure.



Sol: To find V_{th}:

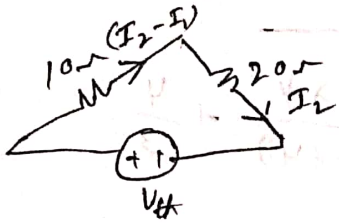


$$(50)I_1 + (-50)I_2 = 10$$

$$(-50)I_1 + (100)I_2 = 0$$

$$I_1 = 0.4A$$

$$I_2 = 0.2A$$

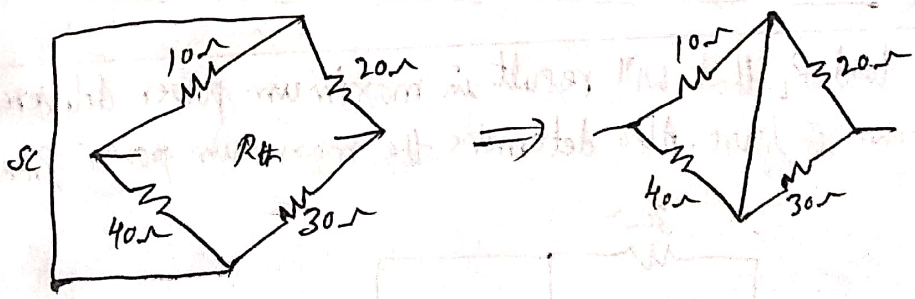


$$10(I_2 - I_1) + 20I_2 - V_{th} = 0$$

$$V_{th} = 10(0.2 - 0.4) + 20 \times 0.2$$

$$V_{th} = 2V$$

To find R_{th}:



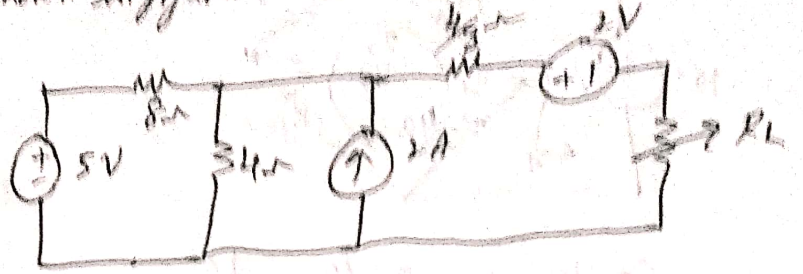
$$R_{th} = \frac{10 \times 40}{(10+40)} + \frac{20 \times 30}{(20+30)} = 8 + 12$$

$$R_{th} = 20Ω$$

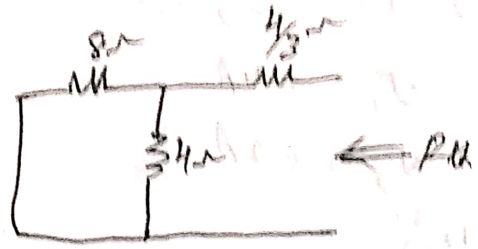
$$R_L = R_{th} = 20Ω$$

$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{2^2}{(4 \times 20)} = 0.05W$$

Q. Find the Value of R_L when maximum power is transferred across it in the network shown in figure.



Sol: To find R_{th}:



$$R_{th} = \frac{8 \times 4}{(8+4)} + \frac{4}{3}$$

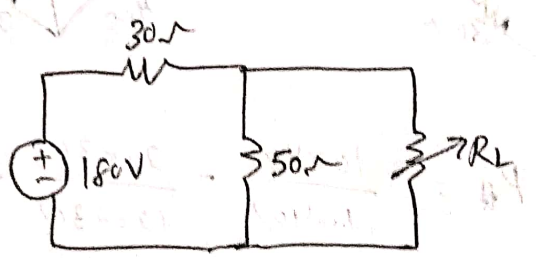
$$R_{th} = 2.66 + 1.33$$

$$R_{th} = 4\Omega$$

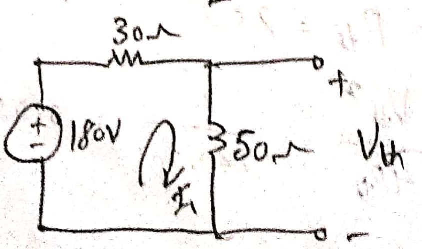
For maximum power transfer, $R_L = R_{th} = 4\Omega$

(No need to find V_{th} since maximum power is not asked in the problem)

Q. Find the load R_L that will result in maximum power delivered to the load for the Circuit shown in figure. Also determine the maximum power P_{max}.



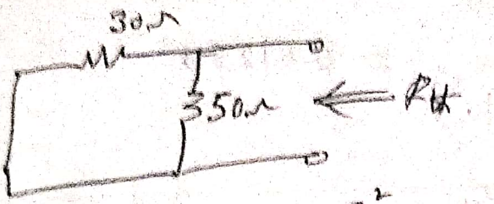
Sol: To find V_{th}: Remove R_L



$$I_1 = \frac{180}{(30+50)} = 2.25A$$

$$V_{th} = 50I_1 = 50 \times 2.25 = 112.5V$$

To find R_{th}:

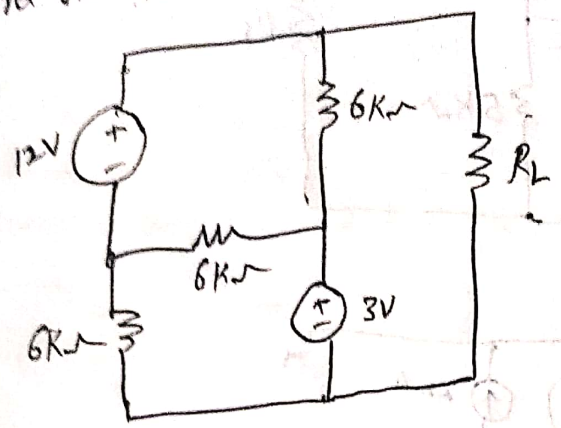


$$R_{th} = \frac{30 \times 50}{(30+50)} = 18.75\Omega$$

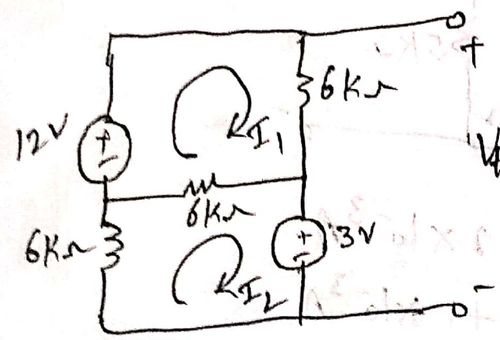
$$R_L = R_{th} = 18.75\Omega$$

$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{112.5^2}{(4 \times 18.75)} = 168.75W$$

3. Refer to the circuit shown in figure, find the value of R_L for maximum power transfer. Also find the maximum power.



A: To find V_{th}:



$$(12K)I_1 + (-6K)I_2 = 12$$

$$(-6K)I_1 + (12K)I_2 = -3$$

$$I_1 = 1.16mA$$

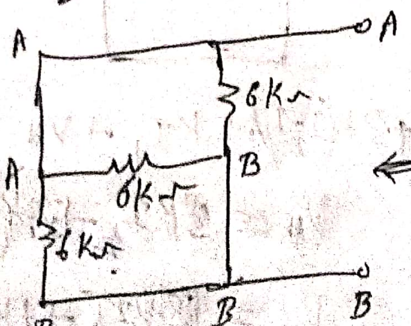
$$I_2 = 3.33 \times 10^{-4}A = 0.333\ mA$$

$$6KI_1 + 3 - V_{th} = 0$$

$$V_{th} = 6 \times 10^3 \times 1.16 \times 10^{-3} + 3$$

$$V_{th} \simeq 10V$$

To find R_{th}:

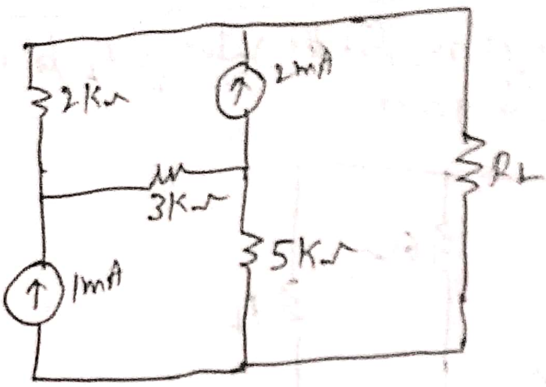


$$R_{th} = \frac{6 \times 10^3}{3} = 2K\Omega \quad \left(\frac{R}{n}\right)$$

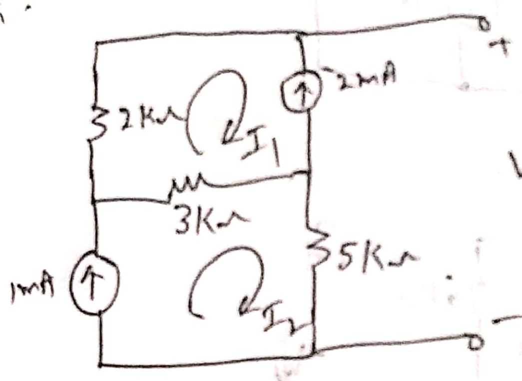
$$R_L = R_{th} = 2K\Omega$$

$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{10^2}{(4\times 2\times 10^3)} = 0.0125\,W$$

Q. Find R_L for maximum power and the maximum power that can be transferred in the network shown in figure.

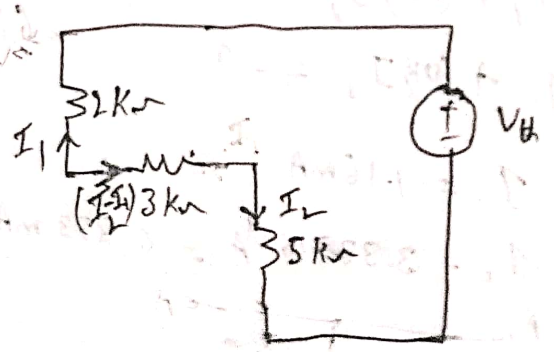


Sol: To find V_{th}:



KCL to non-essential mesh 1: $I_1 = -2\times 10^{-3}A$

KCL to non-essential mesh 2: $I_2 = +1\times 10^{-3}A$



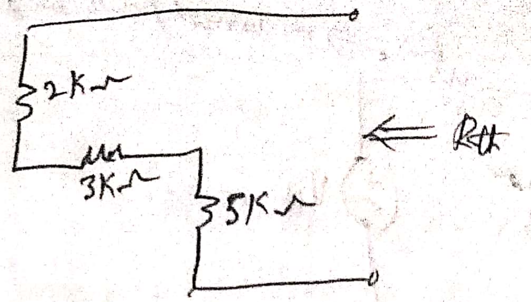
$$-5KI_2 - 3K(I_2 - I_1) + 2KI_1 + V_{th} = 0$$

$$V_{th} = 5KI_2 + 3K(I_2 - I_1) - 2KI_1$$

$$V_{th} = 8KI_2 - 5KI_1$$

$$V_{th} = 8\times 10^3 \times 1\times 10^{-3} - 5\times 10^3 \times -2\times 10^{-3} = 18V$$

To find R_{th}:

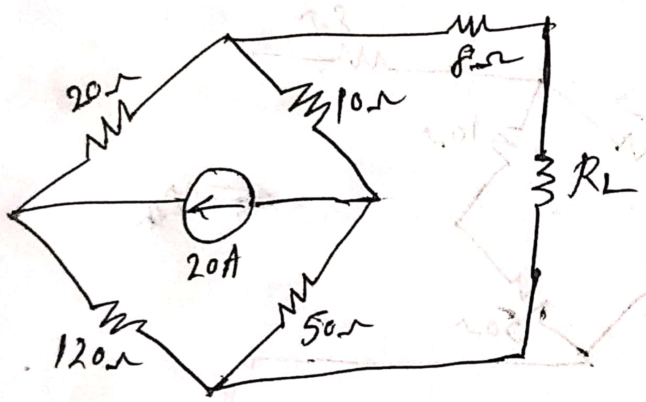


$$R_{th} = 2K + 3K + 5K$$

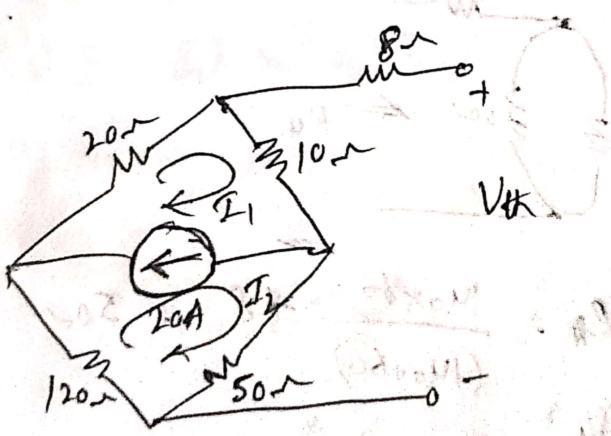
$$R_{th} = 10K\Omega \;, \quad R_L = R_{th} = 10K\Omega$$

$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{18^2}{(4\times 10\times 10^3)} = 8.1mW$$

Q. Find the maximum power dissipated in R_L for the circuit shown in figure.



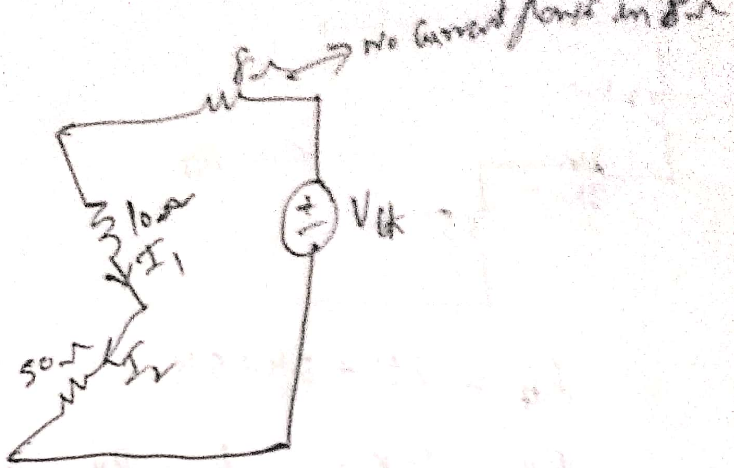
A: To find V_{th}:



KCL for supermesh 1 & 2: $I_1 - I_2 = 20$

KVL for supermesh 1 & 2: $20I_1 + 10I_1 + 50I_2 + 120I_2 = 0$

$$(1)I_1 + (-1)I_2 = 20$$
$$(30)I_1 + (170)I_2 = 0$$
$$I_1 = 17A$$
$$I_2 = -3A$$

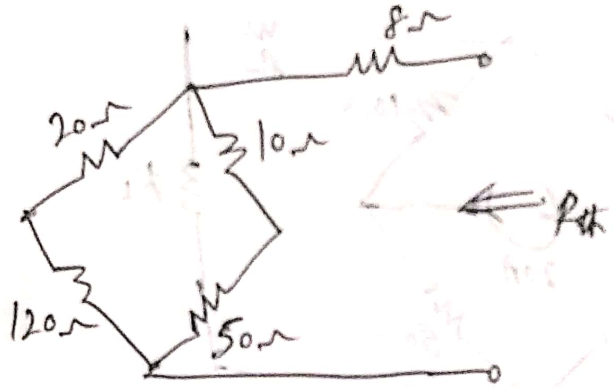


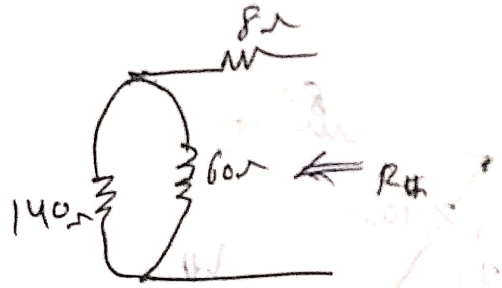
$$10I_1 + 50I_2 - V_{th} = 0$$

$$V_{th} = 10 \times 17 + 50 \times -3$$

$$V_{th} = 20V$$

To find R_{th}:





$$R_{th} = \frac{140 \times 60}{(140+60)} + 8 = 50\Omega$$

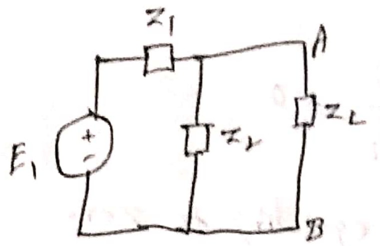
$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{20^2}{(4 \times 50)} = 2W$$

Maximum power transfer theorem (Ac Circuits)

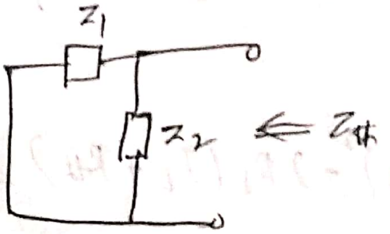
Q: State, explain & prove maximum power transfer theorem.

Sol: Statement: In an active network, maximum power transfer to the load takes place when the load impedance is the complex conjugate of an equivalent impedance of the network as viewed from the terminals of the load.

Explanation:



Let Z_{th} be the equivalent impedance of the network as viewed from the terminals AB and replacing all the independent sources by their internal impedances, as shown in figure below.

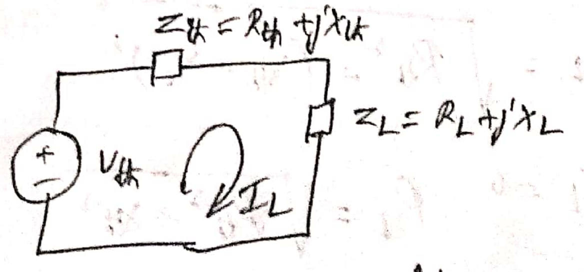


Let this $Z_{th} = R + jx$

Then the maximum power will be transferred to the load, if Z_L is complex conjugate of Z_{th}.

$$Z_L = Z_{th}^* = R - jx$$

Proof:



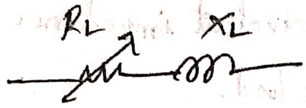
$$I_L = \frac{V_{th}}{(R_{th}+R_L) + j(X_{th}+X_L)}$$

$$|I_L| = \frac{V_{th}}{\sqrt{(R_{th}+R_L)^2 + (X_{th}+X_L)^2}}$$

$$P = |I_L|^2 R_L$$

$$P = \frac{V_{th}^2}{(R_{th}+R_L)^2 + (X_{th}+X_L)^2} \times R_L \quad -①$$

Case i) <u>R_L is Variable</u>, <u>X_L is Constant</u>



$dx = dR_L$
$u = R_L$
$v = (R_{th}+R_L)^2 + (X_{th}+X_L)^2$

Condition for maximum power $\frac{\partial P}{\partial R_L} = 0$

Differentiating ① w.r.t to R_L and equating to zero

$$V_{th}^2 \left[\frac{\{(R_{th}+R_L)^2 + (X_{th}+X_L)^2\} \times 1 - R_L \cdot \{2(R_L+R_{th})\}}{\{(R_{th}+R_L)^2 + (X_{th}+X_L)^2\}^2} \right] = 0$$

$$(R_{th}+R_L)^2 + (X_{th}+X_L)^2 - 2R_L(R_L+R_{th}) = 0$$

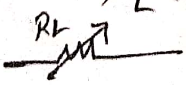
$$R_{th}^2 + R_L^2 + \cancel{2R_LR_{th}} + (X_{th}+X_L)^2 - 2R_L^2 - \cancel{2R_LR_{th}} = 0$$

$$-R_L^2 + R_{th}^2 + (X_{th}+X_L)^2 = 0$$

$$R_L^2 = R_{th}^2 + (X_{th}+X_L)^2$$

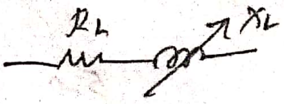
$$R_L = \sqrt{R_{th}^2 + (X_{th}+X_L)^2}$$

When $X_L = 0$ $\quad R_L = \sqrt{R_{th}^2 + X_{th}^2}$



$$R_L = |Z_{th}|$$

Case ii) X_L is variable, R_L is constant



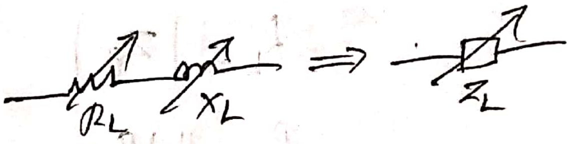
For max. power $\dfrac{\partial P}{\partial X_L} = 0$

$$V_{th}^2 R_L \left[\frac{\{(R_{th}+R_L)^2 + (X_{th}+X_L)^2\} \times 0 - 1\{2(X_{th}+X_L) \cdot 1\}}{\{(R_{th}+R_L)^2 + (X_{th}+X_L)^2\}^2} \right] = 0$$

$$-2(X_{th}+X_L) = 0$$

$$X_L = -X_{th}$$

Case iii) When both are variable

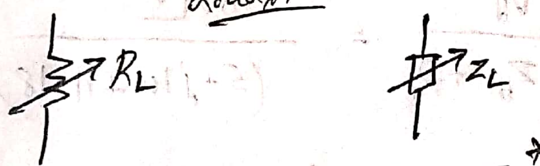


Combining the above equation

$$R_L = R_{th}, \quad X_L = -X_{th}$$

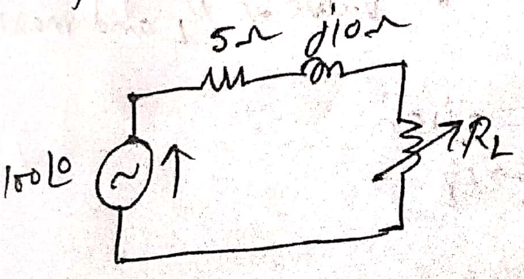
$$Z_L = Z_{th}^*$$

Note : Load side

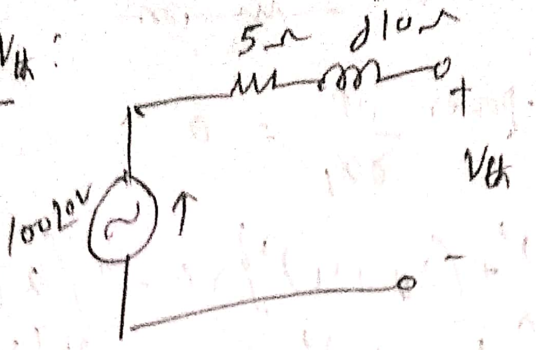


$R_L = |Z_{th}|$ $\qquad$ $Z_L = Z_{th}^*$

Q. In the circuit shown in figure, find the value of R_L which results in maximum power transfer. Calculate the value of the maximum power.

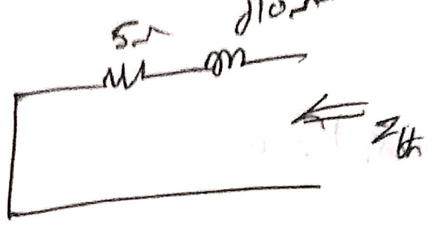


Sol: <u>To find V_{th}:</u>



$V_{th} = 100\angle 0$ V

<u>To find Z_{th}:</u>

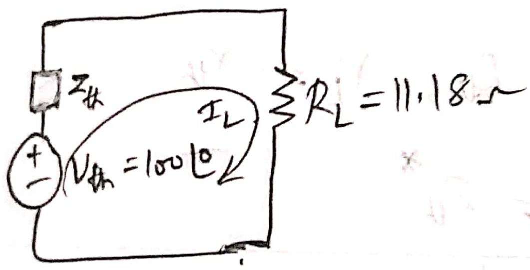


$Z_{th} = 5 + j10$

$Z_{th} = \underbrace{11.18}_{|Z_{th}|}\angle 63.43$

$R_L = |Z_{th}|$

$R_L = 11.18\,\Omega$

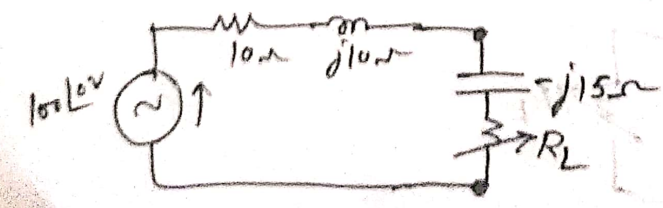


$$I_L = \frac{V_{th}}{Z_{th} + R_L} = \frac{100\angle 0}{(5 + j10 + 11.18)} = \underbrace{5.25}_{|I_L|}\angle -3\ldots$$

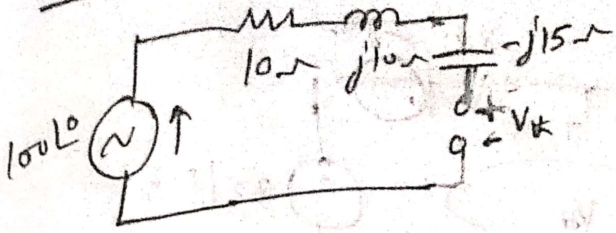
$$P_{max} = |I_L|^2 R_L = 5.25^2 \times 11.18$$

$P_{max} = 308.14$ W

Q. For the network shown in figure determine the value of R_L and maximum po...

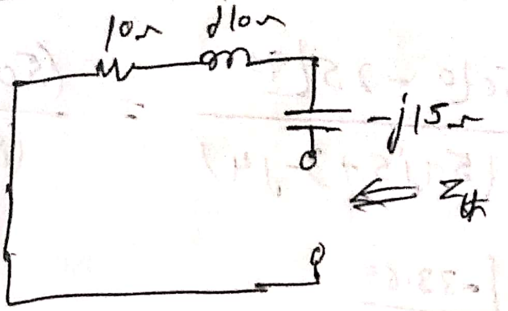


Sol: **To find V_{th}:**



$V_{th} = 100\angle 0 \ V$

To find Z_{th}:

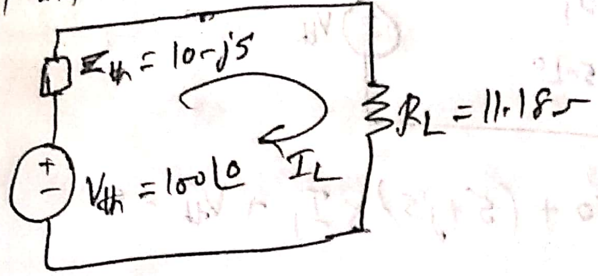


$Z_{th} = 10 + j10 - j15$

$Z_{th} = 10 - j5$

$Z_{th} = \underbrace{11.18}_{|Z_{th}|}\angle -26.56$

$R_L = |Z_{th}| = 11.18\,\Omega$

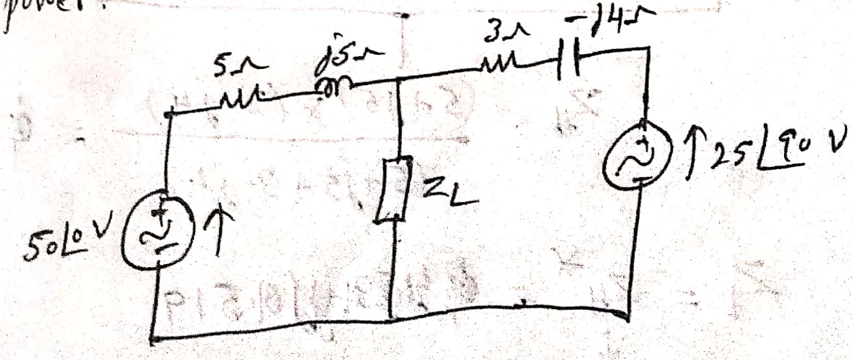


$$I_L = \frac{V_{th}}{Z_{th} + R_L} = \frac{100\angle 0}{(10 - j5 + 11.18)} = \underbrace{4.59}_{|I_L|}\angle 13.28 \ A$$

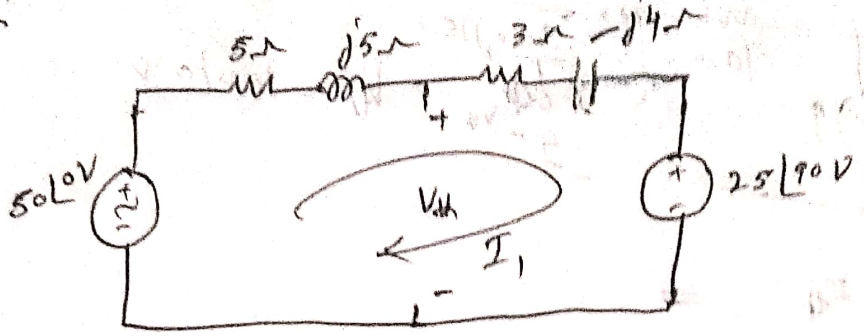
$$P_{max} = |I_L|^2 R_L = 4.59^2 \times 11.18$$

$$P_{max} = 235.54 \ W$$

Q. For the circuit shown in figure find Z_L for maximum power. Also find maximum power.

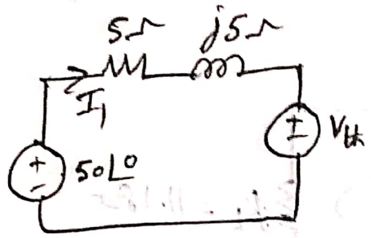


To find V_{th}: Remove Z_L



$$I_1 = \frac{50\angle 0 - 25\angle 90}{(5+j5+3-j4)} = \frac{(50-j25)}{(8+j1)}$$

$$I_1 = 6.93\angle -33.69$$

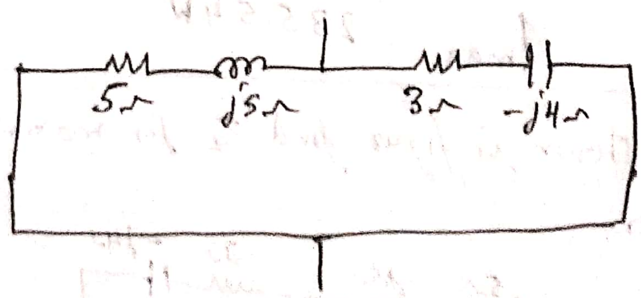


$$-50 + (5+j5) \times I_1 + V_{th} = 0$$

$$V_{th} = 50 - (5+j5) \times 6.93\angle -33.69$$

$$V_{th} = 9.80\angle -78.53$$

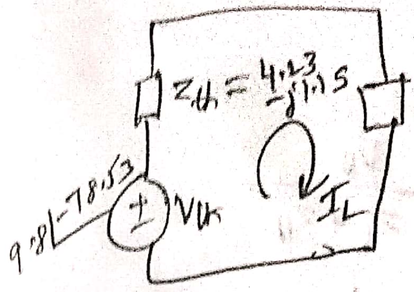
To find Z_{th}:



$$Z_{th} = \frac{(5+j5) \times (3-j4)}{(5+j5+3-j4)} = 4.23 - j1.15$$

$$Z_L = Z_{th}^* = 4.23 + j1.15$$

change the sign of imaginary part of Z_{th}



$$I_L = \frac{V_{th}}{Z_{th} + Z_L} = \frac{9.80\angle{-78.53}}{(4.23 - j1.15 + 4.23 + j1.15)}$$

$$I_L = 1.15\angle{-78.53}$$

$$P_{max} = |I_L|^2 R_L$$

R_L is the real part of Z_L

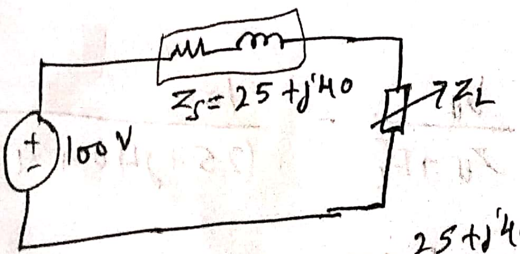
$R_L = 4.23$

$$P_{max} = 1.15^2 \times 4.23$$
$$= 5.59W$$

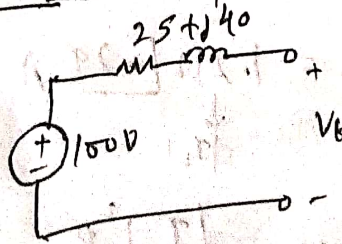
---

3. A source of 100 V feeds a load impedance Z_L through a series impedance $Z_s = (25 + j40)\Omega$ i) Determine the load impedance for maximum power and the value of maximum power ii) if the load consists of pure resistance R_L, find the value of R_L for which maximum power is transferred and maximum power transfer.

i)

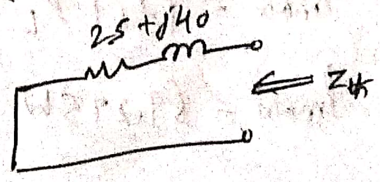


To find V_{th} :



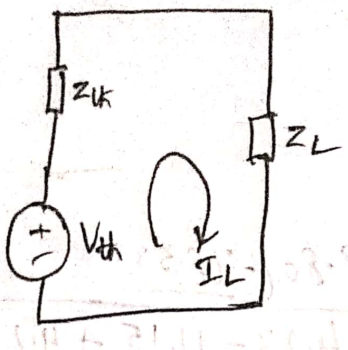
$V_{th} = 100\ V$

To find Z_{th} :



$Z_{th} = 25 + j40$

$$Z_L = Z_{th}^* = 25 - j40$$
$\downarrow$
R_L



$$I_L = \frac{V_{th}}{Z_L + Z_{th}}$$

$$I_L = \frac{100}{(25 - j40 + 25 + j40)}$$

$$I_L = 2\,A$$

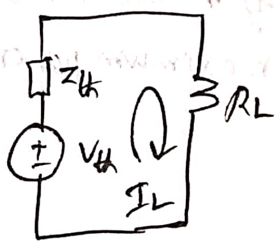
$$P_{max} = |I_L|^2 R_L = 2^2 \times 25 = 100\,W$$

ii) V_{th} is same, $V_{th} = 100\,V$

To find Z_{th}: It is also same

$$Z_{th} = 25 + j40 = \underbrace{47.16}_{|Z_{th}|}\angle 58$$

$$R_L = |Z_{th}| = 47.16\,\Omega$$



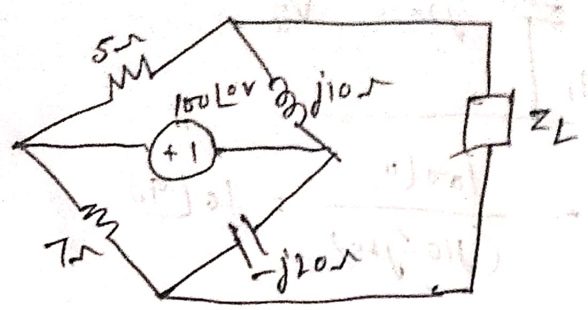
$$I_L = \frac{V_{th}}{Z_{th} + R_L} = \frac{100}{(25 + j40 + 47.16)}$$

$$I_L = \underbrace{1.212}_{|I_L|}\angle -29\ A$$

$$P_{max} = |I_L|^2 R_L = 1.212^2 \times 47.16$$

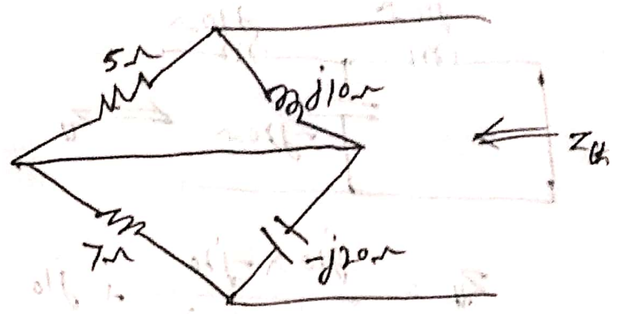
$$P_{max} = 69.275\,W$$

Q. Find the value of Z_L for which maximum power is transferred to the load Z_L from the network shown in figure.



Sol: No need to find V_{th} since maximum power is not asked in the problem.

To find Z_{th} :



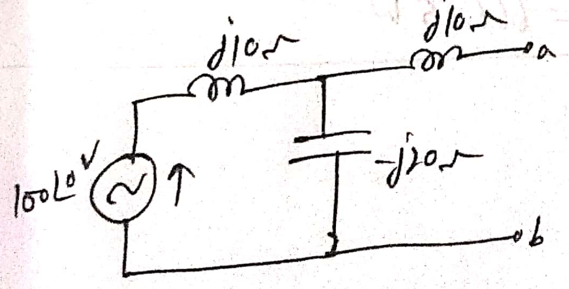
$$Z_{th} = \frac{5 \times j10}{(5+j10)} + \frac{7 \times -j20}{(7-j20)}$$

$$Z_{th} = (4+j2) + (6.23 - j2.182)$$

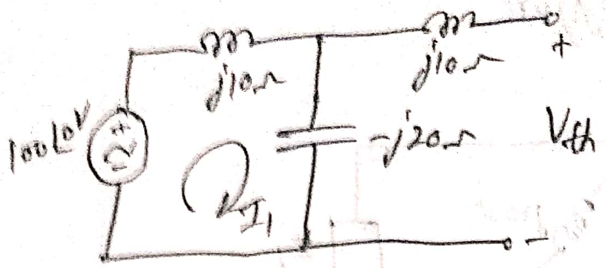
$$Z_{th} = (10.23 - j0.182)\,\Omega$$

$$Z_L = Z_{th}^* = (10.23 + j0.182)\,\Omega$$

Q. What should be the value of pure resistance to be connected across the terminals a & b in the circuit, so that maximum power is transferred to the load. What is the maximum power.



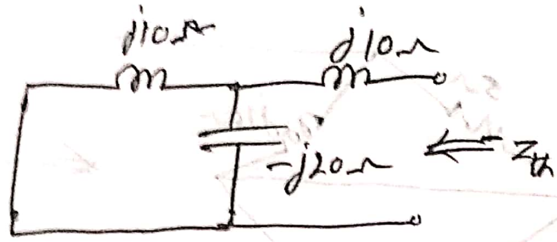
Sol: To find V_{th}



$$I_1 = \frac{100\angle 0}{(j10 - j20)} = 10\angle 90$$

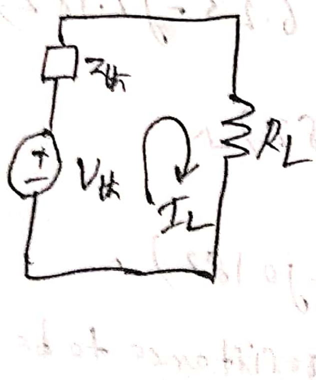
$$V_{th} = -j20 \times I_1 = -j20 \times 10\angle 90 = 200\angle 0\ V$$

To find Z_{th}:



$$Z_{th} = \frac{j10 \times -j20}{(j10 - j20)} + j10 = j30\,\Omega = \underset{|Z_{th}|}{30}\angle 90$$

$$R_L = |Z_{th}| = 30\,\Omega$$
(pure resistance)



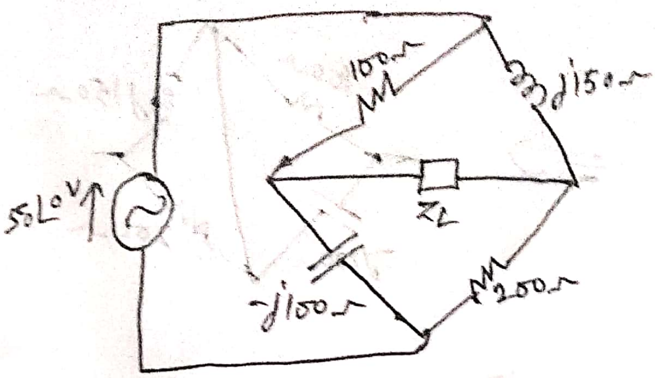
$$I_L = \frac{V_{th}}{Z_{th} + R_L}$$

$$I_L = \frac{200\angle 0}{(j30 + 30)} = 4.714\angle -45$$

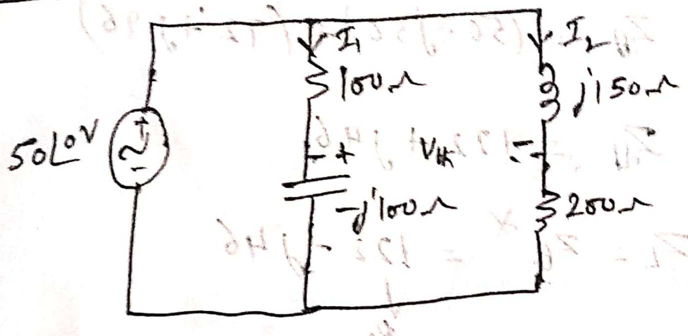
$$P_{max} = |I_L|^2 R_L = 4.714^2 \times 30$$

$$P_{max} = 666.65\ W$$

5. In the network shown in figure, find Z_L so that it takes maximum power and determine the maximum power.

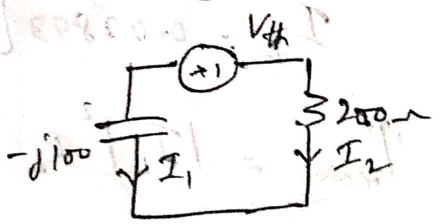


Sol: To find V_{th}:



$$I_1 = \frac{50\angle 0}{(100 - j100)} = 0.3535\angle 45$$

$$I_2 = \frac{50\angle 0}{(j150 + 200)} = 0.2\angle -36.86$$



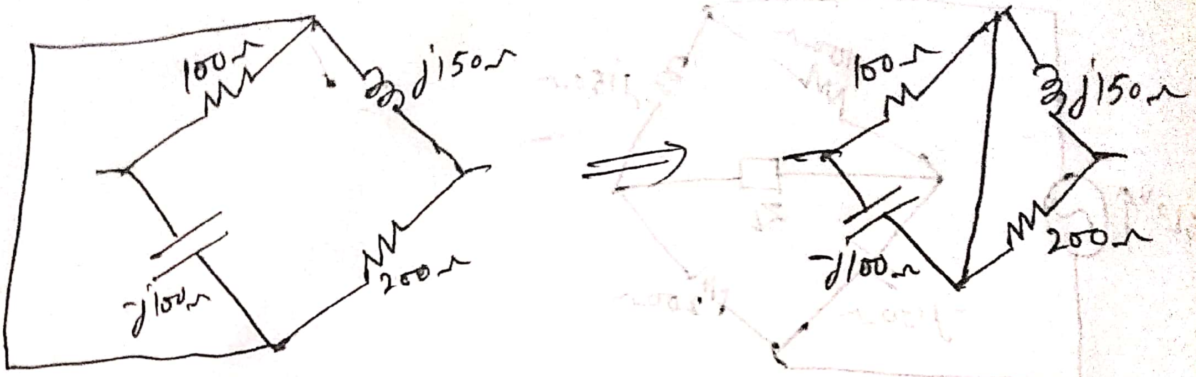
$$+j100 I_1 + V_{th} + 200 I_2 = 0$$

$$V_{th} = -200 I_2 - j100 I_1$$

$$V_{th} = -200 \times 0.2\angle -36.86 - j100 \times 0.3535\angle 45$$

$$V_{th} = 71.079\angle -171.86 \text{ V}$$

To find Z_{th} :

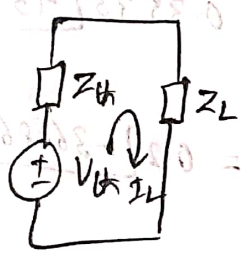


$$Z_{th} = \frac{100 \times -j100}{(100 - j100)} + \frac{200 \times j150}{(200 + j150)}$$

$$Z_{th} = (50 - j50) + (72 + j96)$$

$$Z_{th} = 122 + j46$$

$$Z_L = Z_{th}^* = \underset{\downarrow \atop R_L}{122} - j46$$



$$I_L = \frac{V_{th}}{Z_L + Z_{th}} = \frac{7.07\angle -171.86}{(122 - j46 + 122 + j46)}$$

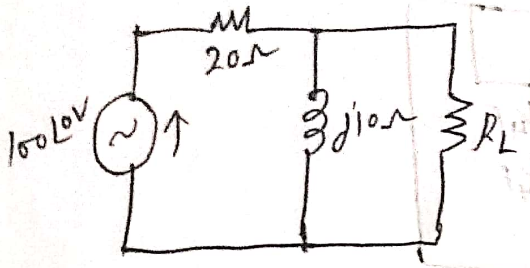
$$I_L = 0.02893\angle -171.86$$

$$P_{max} = |I_L|^2 R_L$$

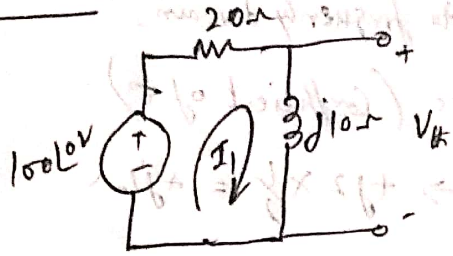
$$= 0.02893^2 \times 122$$

$$P_{max} = 0.102W$$

8. Find the value of R_L of the network shown in figure that will absorb maximum power & specify the value of that power.



Sol: To find V_{th}:

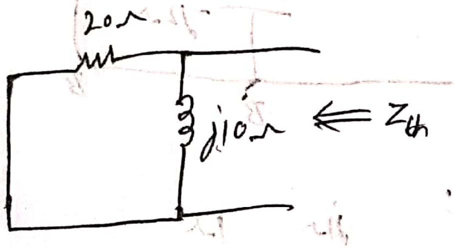


$$I_1 = \frac{100\angle 0}{(20+j10)} = 4.47\angle -26.56$$

$$V_{th} = j10 I_1 = j10 \times 4.47\angle -26.56$$

$$V_{th} = 44.7\angle 63.44$$

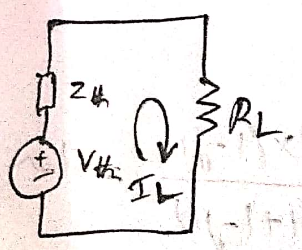
To find Z_{th}:



$$Z_{th} = \frac{20 \times j10}{(20+j10)} = 4+j8$$

$$Z_{th} = \underset{|Z_{th}|}{8.94}\angle 63.43$$

$$R_L = |Z_{th}| = 8.94\,\Omega$$



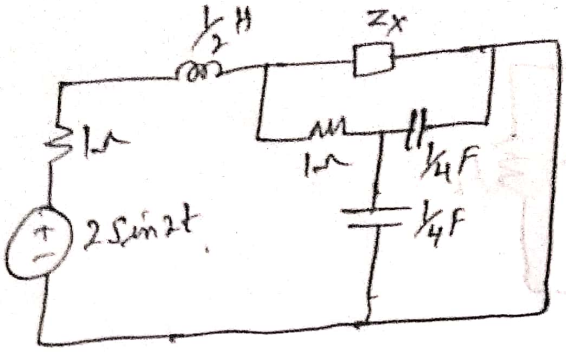
$$I_L = \frac{V_{th}}{Z_{th}+R_L} = \frac{44.7\angle 63.44}{(4+j8+8.94)}$$

$$I_L = 2.93\angle 31.71 \text{ A}$$

$$P_{max} = |I_L|^2 R_L = 2.93^2 \times 8.94$$

$$= 76.74\text{W}$$

Q. For the network shown in figure, determine the impedance Z_x such that maximum power is transferred from the source to the load impedance Z_x.



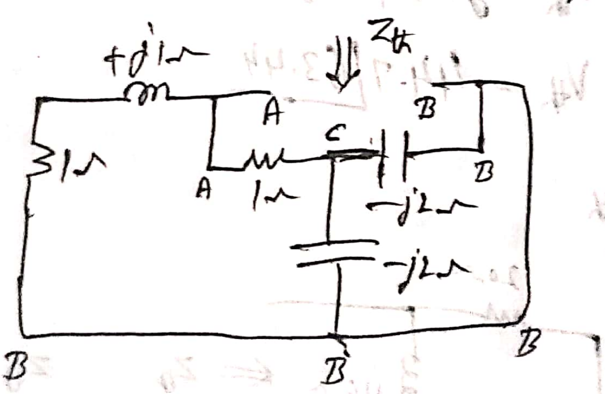
Sol: Convert the time domain circuit to frequency domain,

$$\omega = 2 \text{ rad/sec (Coefficient of } t)$$

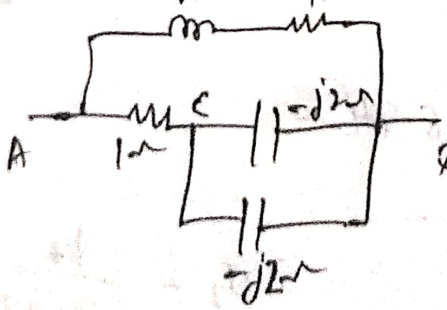
$$\tfrac{1}{2}\text{H } (L) \rightarrow +j\omega L \rightarrow +j2 \times \tfrac{1}{2} = +j1\,\Omega$$

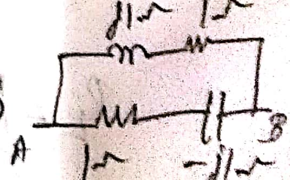
$$\tfrac{1}{4}\text{F } (C) \rightarrow -j \cdot \frac{1}{\omega C} \rightarrow -j \cdot \frac{1}{2 \times \frac{1}{4}} = -j2\,\Omega$$

To find Z_{th}: short circuit the voltage source.



Rewriting the circuit:





$$Z_{th} = Z_{AB} = \frac{(1+j1) \times (1-j1)}{(1+j1+1-j1)} = 1\,\Omega$$

$$Z_x = Z_{th}^{*} = 1\,\Omega$$